Historic Hotels

Topic

Profit model, Optimization, and Problem Solving and Mathematical Reasoning

Key Question

How do you price rooms in a historic hotel to maximize profit?

Learning Goals

Students will:

- Use benefits and constraints to create a procedure to measure profit.
- Consider how to use and exclude data
- Represent real-world situations mathematically
- Make decisions about whether or not a solution meets the needs of a client
- Communicate the solution clearly to the client

Guiding Documents

This activity has the potential to address these and other Grades 6-8 NCTM Mathematics Standards

Numbers and Operations

- work flexibly with fractions, decimals, and percents to solve problems;
- understand and use ratios and proportions to represent quantitative relationships;
- develop meaning for integers and represent and compare quantities with them;
- understand the meaning and effects of arithmetic operations with fractions, decimals, and integers;
- develop, analyze, and explain methods for solving problems involving proportions, such as scaling and finding equivalent ratios.

Algebra

- represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules;
- identify functions as linear or nonlinear and contrast their properties from tables, graphs, or equations;
- model and solve contextualized problems using various representations, such as graphs, tables, and equations.

Problem Solving

- Build new mathematical knowledge through problem solving;
- Solve problems that arise in mathematics and in other contexts;
- Apply and adapt a variety of appropriate strategies to solve problems;
- Monitor and reflect on the process of mathematical problem solving.

Communication

- Organize and consolidate their mathematical thinking through communication;
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- Analyze and evaluate the mathematical thinking and strategies of others;
- Use the language of mathematics to express mathematical ideas precisely.

Connections

- Recognize and use connections among mathematical ideas;
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole;
- Recognize and apply mathematics in contexts outside of mathematics.

Representation

- Create and use representations to organize, record, and communicate mathematical ideas;
- Select, apply, and translate among mathematical representations to solve problems;
- Use representations to model and interpret physical, social, and mathematical phenomena.

Recommended supplies for all MEAs

It is recommended to have all of these supplies in a central location in the room. It is recommended to let the students know that they are available, but not to encourage them to use anything in particular.

- Rulers
- Calculators
- Whiteboards, posterboards, or other presentation tools
- Optional: Computers with programs such as Microsoft Word and Excel

What are Model Eliciting Activities (MEAs)?

Model-Eliciting Activities are problem activities explicitly designed to help students develop conceptual foundations for deeper and higher order ideas in mathematics, science, engineering, and other disciplines. Each task asks students to mathematically interpret a complex real-world situation and requires the

formation of a mathematical description, procedure, or method for the purpose of making a decision for a realistic client. Because teams of students are producing a description, procedure, or method (instead of a one-word or one-number answer), students' solutions to the task reveal explicitly how they are thinking about the given situation.

The Historic Hotels MEA consists of four components:

1) **Background Reading:** Students individually read the newspaper article to become familiar with the context of the problem. This handout is on page 5.

2) **Individual Questions:** Students individually answer these reading comprehension questions about the newspaper article to become even more familiar with the context and beginning thinking about the problem. This handout is on page 6.

3) **Historic Hotels Team Activity:** In teams of three or four, have the teams read through the problem statement and answer the team questions. Then have the teams work on the problem statement for approximately 45 minutes. The time range depends on the amount of self-reflection and revision you want the students to do. It can be shorter if you are looking for students' first thoughts, and can be longer if you expect a polished solution and well-written letter. Each team needs the handout on page 7 and the team questions on the lower half of page 6.

4) **Process of sharing solutions:** Each team writes their solution in a letter or memo to the client. Then, each team presents their solution to the class. Whole class discussion is intermingled with these presentations to discuss the different solutions, the mathematics involved, and the effectiveness of the different solutions in meeting the needs of the client.

In totality, each case study takes approximately 3-5 class periods to implement, but can be shortened by having students do the individual work during out-of-class time. The Presentation Form can be useful and is explained on page 4 and found on page 9.

Recommended Progression of the MEA

Background Reading and Individual Activities: The purpose of the background reading and the individual activities is to introduce the students to the context of the problem. Depending on your instructional purposes, you may want to use a more teacher-directed format or a more student-directed format for going through the background reading and the questions. Some possibilities include:

a. *More teacher-directed* (½ hour): Read the article to the students and give them class time to complete the readiness questions individually. Then, discuss as a class the answers to the readiness questions before beginning work on the problem statement. This approach also works well when you can team with a language arts teacher, and they can go through the article in their class.

b. *More student-directed* (10 minutes): Give the article and the questions to the students the day before for homework. If you wish, you may provide some class time for the students to complete the article and questions. Then, on the day of the case study, discuss as a class the answers to the readiness questions before beginning work on the problem statement.

c. *More student-directed* (10-15 minutes): Give the article and the questions to the students in their teams right before the students begin working on the problem statement. The students answer the questions as a team and then proceed to work on the problem statement.

Working on the Problem Statement (45-90 minutes): Place the students in teams of three or four. If you already use teams in your classroom, it is best if you continue with these same teams since results for MEAs are better when the students have already developed a working relationship with their team members. If you do not use teams in your classroom and classroom management is an issue, the teacher may form

the teams. If classroom management is not an issue, the students may form their own teams. You may want to have the students choose a name for their team to promote unity. Encourage (but don't require or assign) the students to select roles such as timer, collector of supplies, writer of letter, etc. Remind the students that they should share the work of solving the problem. Present the students with the problem statement. Depending on the students' grade level and previous experience with MEAs, you may want to read the problem statement to the students and then identify as a class: a) **the client that the students are working for** and b) **the product that the students are being asked to produce**. Once you have addressed the points above, allow the students to work on the problem statement.

> *Teachers' role:* As they work, your role should be one of a facilitator and observer. Avoid questions or comments that steer the students toward a particular solution. Try to answer their questions with questions so that the student teams figure out their own issues. Also during this time, try to get a sense of how the students are solving the problem so that you can ask them questions about their solutions during their presentations.

Presentations of Solutions (30-45 minutes): The teams present their solutions to the class. There are several options of how you do this. Doing this electronically or assigning students to give feedback as out-of-class work can lessen the time spent on presentations. If you choose to do this in class, which offers the chance for the richest discussions, the following are recommendations for implementation. Each presentation typically takes 3 – 5 minutes. You may want to limit the number of presentations to five or six or limit the number of presentations to the number of original (or significantly different) solutions to the MEA.
Before beginning the presentations, encourage the other students to not only listen to the other teams' presentations but also to a) *try to understand the other teams' solutions* and b) *consider how well these other solutions meet the needs of the client*. You may want to offer points to students that ask 'good' questions of the other teams, or you may want students to complete a reflection page (explanation – page 4, form – page 10) in which they explain how they would revise their solution after hearing about the other solutions. As students offer their presentations and ask questions, whole class discussions should be intermixed with the presentations in order to address conflicts or differences in solutions. When the presentations are over, collect the student teams' memos/letters, presentation overheads, and any other work you would like to look over or assess.

Assessment of Students' Work
You can decide if you wish to evaluate the students' work. If you decide to do so, you may find the following Assessment Guide Rubric helpful:

<u>Performance Level Effectiveness: Does the solution meet the client's needs?</u>
Requires redirection: The product is on the wrong track. Working longer or harder with this approach will not work. The students may need additional feedback from the teacher.

Requires major extensions or refinements: The product is a good start toward meeting the client's needs, but a lot more work is needed to respond to all of the issues.

Requires only minor editing: The product is nearly ready for the client to use. It still needs a few small modifications, additions, or refinements.
Useful for this specific situation: No changes are necessary to meet the client's immediate needs.

Share-able or re-usable: The tool not only works for the immediate solution, but it would be easy for others to modify and use in similar situations. OR The solution goes above and beyond meeting the immediate needs of the client.

Note: If you use this Assessment Guide Rubric for grading purposes, please keep in mind that a performance level of "requires only minor editing" or higher indicates a satisfactory solution. For example, you may want to assign a grade of B for "requires only minor editing", while assigning an A for the next two higher levels. If you give a written score or letter grade after assessing the students' work, we encourage you to provide the students with an explanation (i.e. written comments) as to why they received that score and/or how their solution could be improved. In particular, we found it helpful to phrase the feedback as if it was coming from the client of the problem. So for example, in the historic hotels problem, the client is a hotel owner that wishes to develop a way to figure out his expected profit, and feedback to the students could include statements such as the following:

"I understand what you are saying is the profit for the hotel given this set of information, but I need more information from you about how to apply your procedure when the hotel prices and maintenance costs are different."

Implementing an MEA with Students for the First Time

You may want to let students know the following about MEAs:

- MEAs are longer problems; there are no immediate answers. Instead, students should expect to work on the problem and gradually revise their solution over a period of 45 minutes to an hour.
- MEAs often have more than one solution or one way of thinking about the problem.
- Let the students know ahead of time that they will be presenting their solutions to the class. Tell them to prepare for a 3-5 minute presentation, and that they may use overhead transparencies or other visuals during their presentation.
- Let the students know that you won't be answering questions such as "Is this the right way to do it?" or "Are we done yet?" You can tell them that you will answer clarification questions, but that you will not guide them through the MEA.
- Remind students to make sure that they have returned to the problem statement to verify that they have fully answered the question.
- If students struggle with writing the letter, encourage them to read the letter out loud to each other. This usually helps them identify omissions and errors.

Observing Students as They Work on the Historic Hotels MEA

You may find the Observation Form (page 8) useful for making notes about one or more of your teams of students as they work on the MEA. We have found that the form could be filled out "real-time" as you observe the students working or sometime shortly after you observe the students. The form can be used to record observations about what concepts the students are using, how they are interacting as a team, how they are organizing the data, what tools they use, what revisions to their solutions they may make, and any other miscellaneous comments.

Presentation Form (Optional)

As the teams of students present their solutions to the class, you may find it helpful to have each student complete the presentation form on page 9. This form asks students to evaluate and provide feedback about the solutions of at least two teams. It also asks students to consider how they would revise their own solution to the Historic Hotels MEA after hearing of the other teams' solutions.

Student Reflection Form (Optional)

You may find the Student Reflection Form (page 10) useful for concluding the MEA with the students. The form is a debriefing tool, and it asks students to consider the concepts that they used in solving the MEA and to consider how they would revise their previous solution after hearing of all the different solutions presented by the various teams. Students typically fill out this form after the team presentations. Sometimes students find question #2 confusing, so using this question is optional.

Newspaper Article: An Enchanting Vacation

SAINT PAUL, MN - Going on vacation is something that everyone looks forward to. But staying in a historic hotel transforms any vacation into an enchantment. Finding these charming places is a task to which the National Trust of Historic Hotels of America is committed.

To be recommended by the National Trust of Historic Hotels of America, hotels have to prove that they have faithfully maintained their historic architecture and ambience. Several of these hotels hold great pride in their stories, myths, and legends. For example, in 1856, John Summers invited travelers from all over the world into his home, which eventually became the Greenman House, a 60-room hotel erected in 1871 and destroyed by fire in 1878. By 1878, Mr. Summers and John Baugh, an Eastern hotelier, constructed a much larger and more modern hotel named The Windsor, known at the time to be one of the finest hotels in Saint Paul. The Windsor Hotel was operated successfully until 1880 when Baugh withdrew and sold his interest to Charles J. Monfort. Summers resigned in 1891 and Monfort acted as President and Manager until his death in 1904.

For the next two years the hotel was utilized as an arcade and theater. As the city of Saint Paul continued to grow, the need for a new hotel became increasingly important. In 1908 Lucius P. Ordway, a prime mover in the new hotel project, secured ownership of the property with the intentions of constructing a new luxury hotel. "St. Paul's Million-Dollar Hotel" was opened with much enthusiasm and ceremony on April 18, 1910. The hotel featured a grand ballroom, fine dining room, roof garden, and guestrooms with their own scenic view.

Located in the heart of the city, amid St. Paul's business and cultural districts, this urban hotel overlooks Rice Park and offers guests respite in its tranquil English Cottage Garden. From the hotel guests can walk to St. Paul's theaters, shopping, museums and historic districts, and still hear the horns of the riverboats that ride down the Mississippi River.

Maintenance for a hotel like The Saint Paul Hotel, with all its services, is not an easy task. In 1950, the hotel and city began to suffer as people and businesses moved to the suburbs. The hotel was in need of maintenance and repair, and The Saint Paul Hotel's appeal began to diminish. In 1982, the Saint Paul business community realized the importance of the hotel once again. Piece by piece, the 254 room hotel was redesigned, restored and renovated. Today guests continue to experience the historic European charm and elegance reminiscent of a bygone golden era.

Besides The Saint Paul Hotel, the National Trust of Historic Hotels of America has identified over 140 quality hotels located in 40 states, Canada, and Puerto Rico.

Historic Hotels Model-Eliciting Activity

READINESS QUESTIONS

After reading the "Newspaper Article", answer these questions individually.

1. What do hotels have to accomplish in order to be recommended by the National Trust Historic Hotels of America?

2. What are the main features of The Saint Paul Hotel?

3. Briefly explain the history of the hotels that have stood where The Saint Paul Hotel now stands.

4. What are some responsibilities that a hotel manager might have?

TEAM QUESTIONS

First:

In your team, read the "problem statement".

Second:

In your team. answer these questions:

1. Who are you working for?

2. What do you need to create for him?

3. How will you provide him this information?

Third:

Work together in your team on the problem presented in the "problem statement".

Historic Hotels Model-Eliciting Activity

PROBLEM STATEMENT

Mr. Frank Graham, from St. Paul, has just inherited a historic hotel in town. He would like to keep the hotel, but he has little experience in hotel management. Mr. Graham has asked the St. Paul community for help in making decisions about his new hotel.

The hotel has 80 rooms, and Mr. Graham was told by the previous owner that all of the rooms are occupied when the daily rate is $60 per room. He was also told that for every dollar increase in the daily $60 rate, one less room is rented. So, for example, if he charged $61 dollars per room, only 79 rooms would be occupied. If he charged $62, only 78 rooms would be occupied. Each occupied room has a $4 cost for service and maintenance per day.

Mr. Graham would like to know how much he should charge per room in order to maximize his profit and what his profit would be at that rate. Also, he would like to have a procedure for finding the daily rate that would maximize his profit in the future even if the hotel prices and the maintenance costs change.

Write a letter to Mr. Graham explaining how he can calculate his profit and how much he should charge so that his profit is maximized. Be sure that your method works even if hotel prices and costs rise in the future. Include a good reason for each step of your procedure.

Your team will also be responsible for presenting your solution, procedure, and letter to our class.

Historic Hotels Model-Eliciting Activity

OBSERVATION FORM

Team: _____

Mathematics Concepts Used:
What mathematics concepts and skills did the students use to solve the problem?

Team Interactions:
How did the students interact within their team or share insights with each other?

Data Organization & Problem Perspective:
How did the students organize the problem data? How did the students interpret the task? What perspective did they take?

Tools:
What tools did the students use? How did they use these tools?

Miscellaneous Comments about the team functionality or the problem:

Cycles of Assessment & Justification:
How did the students question their problem-solving processes and their results? How did they justify their assumptions and results? What cycles did they go through?

Historic Hotels Model-Eliciting Activity

PRESENTATION FORM

Name_____

While the presentations are happening, choose TWO teams to evaluate. Look for things that you like about their solution and/or things that you would change in their solution. You are not evaluating their style of presenting. For example, don't write, "They should have organized their presentation better." Evaluate their solution only.

Team _____

Strengths of their solution:

Weaknesses of their solution:

Team _____

Strengths of their solution:

Weaknesses of their solution:

After seeing the other presentations, how would you change your solution? If you would not change your solution, give reasons why your solution does not need changes.

Historic Hotels Model-Eliciting Activity

STUDENT REFLECTION FORM

Name _____ Date_____

1. Please mention the mathematical and scientific "big ideas" and skills (e.g. ratios, proportions, forces, etc.) you used in solving this activity.

2. Think about how related these big ideas and skills are in your solution. In the space below, draw a map or a diagram of the big ideas and skills used in your solution, but arrange the ideas so that those ideas and skills that seem similar are close to each other and those that don't seem related are far apart.

3. After solving this activity, circle the score that best describes how well you understand the mathematical ideas you used.

Not at all A little bit Some Most of it All of it

Explain why you feel that way:

4. How difficult do you think this activity was? Circle your choice.

Easy Little challenging Somewhat challenging Challenging Very Difficult

Explain why you feel that way:

5. After seeing all of your classmates' presentations, what do you think would be the best way for your client, Mr. Graham, to find his daily rate to maximize profit?