Elements característics d' un triangle

Exercicis. Considerem el triangle A(3,1), B(5,3) i C(9,5). Calcula

a) Les tres medianes i el baricentre

$$\begin{aligned} A' &= \frac{B+C}{2} = (7,4) \\ B' &= \frac{A+C}{2} = (6,3) \\ C' &= \frac{A+B}{2} = (4,2) \end{aligned}$$

$m_{BC} \rightarrow \left\{ \begin{array}{c} A(3,1) \\ A'(7,4) \end{array} \right. \rightarrow AA' = A' - A = (4,3)$. Aleshores $\frac{x-3}{4} = \frac{y-1}{3} \rightarrow$ 3x-4y=5.

$m_{AC} \rightarrow \left\{ \begin{array}{c} B(5,3) \\ B'(6,3) \end{array} \right. \rightarrow BB' = B' - B = (1,0)$. Aleshores $\frac{x-5}{1} = \frac{y-3}{0} \rightarrow$ y=3.

$m_{AB} \rightarrow \left\{ \begin{array}{c} C(9,5) \\ C'(4,2) \end{array} \right. \rightarrow CC' = C' - C = (-5,-3)$. Aleshores $\frac{x-4}{-5} = \frac{y-2}{-3} \rightarrow$ -3x+5y=-2.

Per tant, el baricentre serà $\left\{ \begin{array}{c} y = 3 \\ 3x - 4y = 5 \end{array} \right. \rightarrow$ 3x-4·3=5$\rightarrow$ x=$\frac{17}{3} \rightarrow (\frac{17}{3}, 3)$

b) Les tres mediatrius i el circumcentre

$$\begin{aligned} A' &= \frac{B+C}{2} = (7,4) \\ B' &= \frac{A+C}{2} = (6,3) \\ C' &= \frac{A+B}{2} = (4,2) \end{aligned}$$

A(3,1), B(5,3) i C(9,5)

$med_{BC} \rightarrow \left\{ \begin{array}{c} BC = (4,2) \rightarrow \text{ (-2,4)} \\ A'(7,4) \end{array} \right.$. Aleshores $\frac{x-7}{-2} = \frac{y-4}{4} \rightarrow$ 4x+2y=36.

$med_{AC} \rightarrow \left\{ \begin{array}{c} AC = (6,4) \rightarrow \text{ (-4,6)} \\ B'(6,3) \end{array} \right.$. Aleshores $\frac{x-6}{-4} = \frac{y-3}{6} \rightarrow$ 6x+4y=48.

$m_{AB} \rightarrow \left\{ \begin{array}{c} AB = (2,2) \rightarrow \text{ (-2,2)} \\ C'(4,2) \end{array} \right.$. Aleshores $\frac{x-4}{-2} = \frac{y-2}{2} \rightarrow$ 2x+2y=12.

Per tant, el circumcentre serà $\left\{\begin{array}{c} 2x+2y=12 \\ 4x+2y=36 \end{array}\right. \rightarrow \left\{\begin{array}{c} x+y=6 \\ 2x+6-x=36 \rightarrow x=30 \end{array}\right. \rightarrow$ y=-24 i, per tant, (30,-24).

c) b) Les tres alçades i l' ortocentreA(3,1), B(5,3) i C(9,5)

$h_{BC} \rightarrow \left\{\begin{array}{c} BC=(4,2) \rightarrow \text{ (-2,4)} \\ A(3,1) \end{array}\right.$. Aleshores $\frac{x-3}{-2}=\frac{y-1}{4} \rightarrow$ 4x+2y=14.

$h_{AC} \rightarrow \left\{\begin{array}{c} AC=(6,4) \rightarrow \text{ (-4,6)} \\ B(5,3) \end{array}\right.$. Aleshores $\frac{x-5}{-4}=\frac{y-3}{6} \rightarrow$ 6x+4y=42.

$m_{AB} \rightarrow \left\{\begin{array}{c} AB=(2,2) \rightarrow \text{ (-2,2)} \\ C(9,5) \end{array}\right.$. Aleshores $\frac{x-9}{-2}=\frac{y-5}{2} \rightarrow$ 2x+2y=28.

Per tant, l' ortocentre serà $\left\{\begin{array}{c} 2x+2y=28 \\ 4x+2y=14 \end{array}\right. \rightarrow \left\{\begin{array}{c} x+y=14 \\ 2x+14-x=7 \rightarrow x=-7 \end{array}\right. \rightarrow$ y=21 i, per tant, (-7,21).