Exercicis deTrigonometria

Exercici 1. *Si sina* $= \frac{1}{5}$, *90º<a* < 180 i *cosb* $= \frac{1}{3}$ $270^o < b < 360^o$. *Troba cos(a − b), sin(a + b), tag2a i sin* $\frac{a}{2}$.

En efecte,

$$(\frac{1}{5})^2 + \cos^2 a = 1$$

$$\frac{1}{25} + \cos^2 a = 1$$

$$\cos^2 a = 1 - \frac{1}{25} = \frac{24}{25}$$

$$\cos a = -\sqrt{\frac{24}{25}} = \frac{-\sqrt{24}}{5}$$

$$(\frac{1}{3})^2 + \sin^2 b = 1$$

$$\frac{1}{9} + \sin^2 a = 1$$

$$\sin^2 b = 1 - \frac{1}{9} = \frac{8}{9}$$

$$\sin b = -\sqrt{\frac{8}{9}} = \frac{-\sqrt{8}}{3}$$

$$\sin(a+b) = \sin a \cos b + \cos a \sin b = \frac{1}{5}\cdot\frac{1}{3} + (\frac{-\sqrt{24}}{5})\frac{-\sqrt{8}}{3} = \frac{1+\sqrt{192}}{15}$$

$$\cos(a-b) = \cos a \cos b + \sin a \sin b = (\frac{-\sqrt{24}}{5})\cdot\frac{1}{3} + (\frac{1}{5})(\frac{-\sqrt{8}}{3}) = \frac{-\sqrt{24}-\sqrt{8}}{15}$$

$$tag2a = \frac{\sin 2a}{\cos 2a} = \frac{2\sin a \cos a}{\cos^2 a - \sin^2 a} = \frac{2\frac{1}{5}\frac{-\sqrt{24}}{5}}{(\frac{-\sqrt{24}}{5})^2 - (\frac{1}{5})^2} = \frac{2\sqrt{24}}{23}$$

$$\sin\frac{a}{2} = \sqrt{\frac{1-\cos a}{2}} = \sqrt{\frac{1+\frac{\sqrt{24}}{5}}{2}} = \sqrt{\frac{5+\sqrt{24}}{2}}$$

Exercici 2. *Una persona vol saber a quina distància estan dos arbres. Per això es situa a 25 metres d' un dels arbres i a 30 metres de l'altre. L'angle que forma aquesta senyor amb els dos arbres es de 50º. A quina distància es troben els arbres?*

En efecte,

$$c^2 = b^2 + a^2 - 2ba\cos C$$
$$c^2 = 25^2 + 30^2 - 2{\cdot}25{\cdot}30\cos 50$$
$$c^2 = 1525 - 1500\cos 50$$
$$c^2 = 560,82 \rightarrow c = 23,68$$

Exercici 3. *Demostra que*

$$tag^2 a = \sec^2 a - 1$$
$$\cot ag^2 a = co\sec^2 a - 1$$

En efecte,

$$tag^2 a = \frac{\sin^2 a}{\cos^2 a} = \frac{1-\cos^2 a}{\cos^2 a} = \frac{1}{\cos^2 a} - \frac{\cos^2 a}{\cos^2 a} = \frac{1}{\cos^2 a} - 1 = \sec^2 a - 1$$
$$\cot ag^2 a = \frac{\cos^2 a}{\sin^2 a} = \frac{1-\sin^2 a}{\sin^2 a} = \frac{1}{\sin^2 a} - \frac{\sin^2 a}{\sin^2 a} = co\sec^2 a - 1$$