Integració

Definició. Siguin $F(x)$ i $f(x)$ dues funcions definides en un interval $[a, b]$. Aleshores direm que $F(x)$ és una **primitiva** de $f(x)$ si $F\prime(x) = f(x)$. Així, per exemple, podem dir que,

- x^2 és una primitiva de $2x$ ja que $(x^2)\prime = 2x$.
- x^2-5 és una primitiva de $2x$ ja que $(x^2 - 5)\prime = 2x$.
- lnx és una primitiva de $\frac{1}{x}$ ja que $(lnx)\prime = \frac{1}{x}$.
- arctagx és una primitiva $\frac{1}{1+x^2}$ de $(arctagx)\prime=\frac{1}{1+x^2}$.

Hem vist que la primitiva d'una funció no és única de fet si $y = F(x)$ és una primitiva de $f(x)$ aleshores $g(x) = F(x) + C$ també ho és.

En efecte, com que $F(x)$ és una primitiva de $f(x)$ tindrem que $(F(x))\prime = f(x)$ i alehsores $(F(x) + C)\prime = f(x) + 0 = f(x)$ i per tant $F(x) + C$ és una primitiva de $f(x)$.

Les principals propietats lineals de les integrals són,

- $\int (f(x) \pm g(x))\ dx = \int f(x)dx \pm \int g(x)dx$ ja que $(f(x) \pm g(x))\prime = (f(x))\prime + (g(x))\prime$.
- $\int k{\cdot}f(x)dx = k{\cdot}\int f(x)dx$.

Llista d'integrals immediates

- $\int x^n dx=\frac{x^{n+1}}{n+1} + C$ ja que $(\frac{x^{n+1}}{n+1}+C)' = x^n$.
- $\int \frac{1}{x}dx = lnx + C$ ja que $(lnx + C)\prime = \frac{1}{x}$.
- $\int e^x dx = e^x + C$ ja que $(e^x+C)' = e^x$.
- $\int a^x dx=\frac{a^x}{lna} + C$ ja que $(\frac{a^x}{lna}+C)'=a^x$.
- $\int sinxdx = -cosx + C$ ja que $(-cosx + C)\prime = sinx$.
- $\int cosxdx = sinx + C$ ja que $(sinx + C)\prime = cosx$.

- $\int \frac{1}{\cos^2 x}\ dx = tagx + C$ ja que $(tagx + C)\prime = \frac{1}{\cos^2 x}$.
- $\int \frac{1}{\sqrt{1-x^2}}dx = arcsinx + C$ ja que $(arcsinx + C)\prime = \frac{1}{\sqrt{1-x^2}}$.
- $\int \frac{-1}{\sqrt{1-x^2}}dx = arccosx + C$ ja que $(arccosx + C)\prime = \frac{-1}{\sqrt{1-x^2}}$.
- $\int \frac{1}{1+x^2}dx = arctagx + C$ ja que $(arctagx + C)\prime = \frac{1}{1+x^2}$.

A més a més si apliquem la regla de la cadena obtindrem:

- $\int (f(x))^n \cdot f\prime(x)dx = \frac{(f(x))^{n+1}}{n+1} + C$ ja que $(\frac{(f(x))^{n+1}}{n+1} + C)' = (f(x))^n \cdot f\prime(x)$.
- $\int \frac{1}{f(x)} \cdot f\prime(x)dx = lnf(x) + C$ ja que $(lnf(x) + C)\prime = \frac{1}{f(x)} \cdot f\prime(x)$.
- $\int e^{f(x)} f\prime(x)dx = e^{f(x)} + C$ ja que $(e^{f(x)} + C)\prime = e^{f(x)} f\prime(x)$.
- $\int a^{f(x)} \cdot f\prime(x)dx = \frac{a^{f(x)}}{lna} + C$ ja que $(\frac{a^{f(x)}}{lna} + C)' = a^{f(x)} \cdot f\prime(x)$.
- $\int sinf(x) \cdot f\prime(x)dx = -cosf(x) + C$ ja que $(-cosf(x) + C)\prime = sinf(x) \cdot f\prime(x)$.
- $\int cosf(x) \cdot f\prime(x)dx = sinf(x) + C$ ja que $(sinf(x) + C)\prime = cosf(x) \cdot f\prime(x)$.
- $\int \frac{1}{cos^2 f(x)} \cdot f\prime(x)dx = tagf(x) + C$ ja que $(tagf(x) + C)\prime = \frac{1}{cos^2 f(x)} \cdot f\prime(x)$.
- $\int \frac{1}{\sqrt{1-f(x)^2}} \cdot f\prime(x)dx = arcsinf(x) + C$ ja que $(arcsinf(x) + C)\prime = \frac{1}{\sqrt{1-f(x)^2}} \cdot f\prime(x)$.
- $\int \frac{-1}{\sqrt{1-(f(x))^2}} \cdot f\prime(x)dx = arccosf(x) + C$ ja que $(arccosf(x) + C)\prime = \frac{-1}{\sqrt{1-(f(x))^2}} \cdot f\prime(x)$.
- $\int \frac{1}{1+(f(x))^2} \cdot f\prime(x)dx = arctagf(x) + C$ ja que $(arctagf(x) + C)\prime = \frac{1}{1+(f(x))^2} \cdot f\prime(x)$.

Exemples

- $\int x^3 + 5x^2 - 2x + 3\ dx = \frac{x^4}{4} + 5\frac{x^3}{3} - x^2 + 3x + C$.
- $\int 2x^4 - 6x^3 - 3x + 2\ dx = 2\frac{x^5}{5} - 6\frac{x^4}{4} - 3\frac{x^2}{2} + 2x + C$.
- $\int (\sqrt[5]{x^4} + 2x^3 - \frac{4}{x^3})\ dx = \int (x^{\frac{4}{5}} + 2x^3 - 4x^{-3})\ dx = \frac{x^{\frac{4}{5}+1}}{\frac{4}{5}+1} + \frac{2x^4}{4} - \frac{x^{-2}}{-2} + C = \frac{5\sqrt[5]{x^9}}{9} + \frac{x^4}{2} + \frac{x^2}{2} + C$.
- $\int \sin x + 2\cos x - e^x\ dx = -\cos x + 2\sin x - e^x + C$.

- $\int \frac{3}{1+x^2} + \frac{4}{\sqrt{1-x^2}} - 3\ dx = 3arctagx x + 4 \arcsin x - 3x + C.$
- $\int 4^x + 2^x + \frac{1}{x}\ dx = \frac{4^x}{\ln 4} + \frac{2^x}{\ln 2} + \ln x + C.$
- $\int (2x-3)^{11} dx = \frac{1}{2} \int (2x-3)^{11} 2dx = \frac{(2\mathrm{x}\text{-}3)^{12}}{24} + C.$
- $\int (2e^{\mathrm{x}} - 3)^{11} e^{\mathrm{x}} dx = \frac{1}{2} \int (2e^{\mathrm{x}} - 3)^{11} 2e^{\mathrm{x}} dx = \frac{(2\mathrm{e}^{\mathrm{x}}\text{-}3)^{12}}{24} + C.$
- $\int \frac{(\mathrm{lnx}+5)^4}{\mathrm{x}} dx = \frac{(\mathrm{lnx}+5)^5}{5} + C.$
- $\int \frac{3}{\mathrm{x}}\ \mathrm{dx}{=}3\int \frac{1}{\mathrm{x}}\ \mathrm{dx}{=}3lnx + C.$
- $\int \frac{3}{\mathrm{x}+2} dx = 3 \int \frac{1}{\mathrm{x}+2}\ dx = 3ln(x+2) + C.$
- $\int \frac{\mathrm{x}^2}{\mathrm{x}^3+2}\ \mathrm{dx}{=}\frac{1}{3} \int \frac{x^2}{\mathrm{x}^3+2}\ dx = \frac{1}{3} ln(x^3+2) + C.$
- $\int \frac{\mathrm{cosx}}{sinx+8}\ \mathrm{dx}{=}\int \frac{\mathrm{cosx}}{\mathrm{sinx}+8}\ \mathrm{d}x = ln(sinx+8) + C.$
- $\int e^{\mathrm{x}} dx = e^{\mathrm{x}} + C.$
- $\int e^{3\mathrm{x}\text{-}1} dx = \frac{1}{3} \int e^{3\mathrm{x}\text{-}1} \cdot 3dx = \frac{1}{3} \cdot e^{3\mathrm{x}\text{-}1} + C.$
- $\int \frac{\mathrm{e}^{\mathrm{arcsinx}}}{\sqrt{1\text{-}\mathrm{x}^2}}\ \mathrm{dx}{=}e^{\mathrm{arcsinx}} + C.$
- $\int sinxdx = -cosx + C.$
- $\int e^{\mathrm{sinx}} \cdot cosxdx = e^{\mathrm{sinx}} + C.$
- $\int \frac{\mathrm{e}^{\sqrt{\mathrm{x}}}}{\sqrt{\mathrm{x}}}\ \mathrm{dx}{=}2 \cdot \int \frac{\mathrm{e}^{\sqrt{\mathrm{x}}}}{2\sqrt{\mathrm{x}}}\ \mathrm{dx}{=}\ 2e^{\sqrt{\mathrm{x}}}{+}\mathrm{C}.$
- $\int sin(lnx) \cdot \frac{1}{x} dx = -cos(lnx) + C.$
- $\int sine^{\mathrm{x}} \cdot e^{\mathrm{x}} dx = -cose^{\mathrm{x}} + C.$
- $\int cosxdx = sinx + C.$
- $\int \cos(x^2+1) \cdot xdx = \frac{1}{2} \int cos(x^2+1) \cdot 2xdx = \frac{1}{2}(sin(x^2+1)) + C.$
- $\int \frac{1\ \mathrm{dx}}{\sqrt{1\text{-}4\mathrm{x}^2}} = \frac{1}{2} \int \frac{2\ \mathrm{dx}}{\sqrt{1\text{-}(2\mathrm{x})^2}} = \frac{1}{2} \cdot arcsin(2x) + C.$
- $\int \frac{\mathrm{x\ dx}}{\sqrt{1\text{-}4\mathrm{x}^4}} = \frac{1}{4} \int \frac{4\mathrm{x\ dx}}{\sqrt{1\text{-}(2\mathrm{x}^2)^2}} = \frac{1}{4} \cdot arcsin(2x^2) + C.$
- $\int \frac{\mathrm{x\ dx}}{\sqrt{3\text{-}4\mathrm{x}^4}} = \frac{1}{\sqrt{3}} \cdot \int \frac{\mathrm{x\ dx}}{\sqrt{1\text{-}\frac{4}{3}\mathrm{x}^4}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{4} \int \frac{\frac{4}{\sqrt{3}}\mathrm{x\ dx}}{\sqrt{1\text{-}(\frac{2}{\sqrt{3}}\mathrm{x}^2)^2}} = \frac{1}{4} \cdot \mathrm{arcsin}(\frac{2}{\sqrt{3}}\mathrm{x}^2){+}\mathrm{C}.$

- $\int \frac{-e^x\ dx}{\sqrt{1\text{-}e^{2x}}} = \int \frac{-e^x\ dx}{\sqrt{1\text{-}(e^x)^2}} = \text{arcos}e^x + C.$
- $\int \frac{3}{1+2x^2} dx = 3\cdot\int \frac{1}{1+(\sqrt{2}x)^2} dx = \frac{1}{\sqrt{2}}\cdot\int \frac{\sqrt{2}}{1+(\sqrt{2}x)^2} dx = \frac{1}{\sqrt{2}}\cdot arctag(\sqrt{2}x) + C.$
- $\int \frac{\frac{1}{x}}{1+(\ln x)^2}\ dx = arctag(lnx) + C.$
- $\int \frac{5}{3+4x^2}\ dx = 5\cdot\frac{1}{3}\cdot\int \frac{1}{1+\frac{4}{3}x^2}\ dx = \frac{5}{3}\cdot\frac{\sqrt{3}}{2}\cdot\int \frac{\frac{2}{\sqrt{3}}}{1+(\frac{2}{\sqrt{3}}x)^2}\ dx = \frac{5}{3}\cdot\frac{\sqrt{3}}{2}\cdot\text{arctag}(\frac{2}{\sqrt{3}}x) + C.$

Exercicis resolts

Exercici 1. Troba una funció f(x) la derivada de la qual sigui f'(x)=4x+3 i tal que f(2)=3.

- $f(x) = \int 4x+3\ dx = 2x^2 + 3x + C$. Per tant totes les funcions del món tals que f'(x)=4x+3 són de la forma $f(x)=2x^2+3x+C$ i nosaltres en busquem una tal que f(2)=3. Per tant $2\cdot2^2 + 3\cdot2 + C = 8+6+C = 14+C = 3 \rightarrow c = -11$ i la funció serà $f(x) = 2x^2 + 3x - 11$.

Exercici 2. D'entre totes les funcions tals que F(x) tals que $F'(x) = \frac{x}{x^2+1}$ troba la que F(0)=5.

- $F(x) = \int \frac{x}{x^2+1}\ dx = \ln(x^2+1) + C \rightarrow \ln 1 + C = 5 \rightarrow C = 5$ i per tant $F(x) = \ln(x^2+1) + 5$.

Exercici 3. Resol les següents integrals $\int (x^3 + \frac{3}{x^2} + \frac{2}{\sqrt[3]{x}})\ dx$, $\int (x^3 + 2x^2 + 4x + \frac{1}{x})$ dx.

- $\int (x^3 + \frac{3}{x^2} + \frac{2}{\sqrt[3]{x}})\ dx = \int (x^3 + 3\cdot x^{-2} + 2\cdot x^{\frac{-1}{3}})\ dx = \frac{x^4}{4} + 3\cdot\frac{-1}{x} + 2\cdot\frac{x^{\frac{2}{3}}}{\frac{2}{3}} + C = \frac{x^4}{4} - \frac{3}{x} + 3\cdot\sqrt[3]{x^2} + C.$
- $\int (x^3 + 2x^2 + 4x + \frac{1}{x})\ dx = \frac{x^4}{4} + 2\cdot\frac{x^3}{3} + 4\cdot\frac{x^2}{2} + \ln x + C = \frac{x^4}{4} + 2\cdot\frac{x^3}{3} + 2\cdot x^2 + \ln x + C.$

Exercici 4. Resol les següents integrals $\int \frac{(2\ln x+3)^4}{x}\ dx$, $\int \sqrt{3x^2+5}\cdot x\ dx$, $\int \frac{dx}{\sqrt[3]{3x+5}}$.

- $\int \frac{(2\ln x+3)^4}{x}\ dx = \frac{1}{2}\cdot\int \frac{(2\ln x+3)^4}{x}\cdot 2\ dx = \frac{1}{2}\cdot\frac{(2\ln x+3)^5}{5} + C.$

- $\int\sqrt{3x^2+5}\cdot x = \frac{1}{2}\int(3x^2+5)^{\frac{1}{2}}\cdot 2x\ dx = \frac{1}{2}\cdot\frac{(3x^2+5)^{\frac{3}{2}}}{\frac{3}{2}}+C=\frac{(3x^2+5)^{\frac{3}{2}}}{3}+C.$

- $\int\frac{dx}{\sqrt[3]{3x+5}} = \frac{1}{3}\cdot\int\ (3x+5)^{\frac{-1}{3}}\cdot 3\ dx=\frac{1}{3}\cdot\frac{(3x+5)^{\frac{2}{3}}}{\frac{2}{3}}+C=\frac{(3x+5)^{\frac{2}{3}}}{2}+C.$

Exercici 5. Resol les següents integrals $\int\ \frac{1}{\sqrt{1-x^2}\cdot\text{arcsinx}}$ dx, $\int\ \frac{8x}{7x^2+2}$ dx, $\int\ \frac{\sin x}{\cos x+3}$ dx.

- $\int\ \frac{1}{\sqrt{1-x^2}\cdot\text{arcsinx}}$ dx= ln(arcsinx)+C.
- $\int\ \frac{8x}{7x^2+2}$ dx=$8\cdot\frac{1}{14}\cdot\int\ \frac{14x}{7x^2+2}$ dx=$\frac{4}{7}\cdot\ln(7x^2+2)$+C.
- $\int\ \frac{\sin x}{\cos x+3}$ dx=ln(cosx+3)+C.

Exercici 6. Resol les següents integrals $\int e^{x^2+3}\cdot x$ dx, $\int\ \frac{e^{\frac{1}{x}}}{\frac{1}{x^2}}$ dx, $\int e^{5x-3}$ dx.

- $\int e^{x^2+3}\cdot x\ dx=\frac{1}{2}\cdot\ \int e^{x^2+3}\cdot 2x\ dx=\frac{1}{2}\cdot e^{x^2+3}+C.$
- $\int\ \frac{e^{\frac{1}{x}}}{\frac{1}{x^2}}\ dx=-\int\ \frac{e^{\frac{1}{x}}}{-\frac{1}{x^2}}\ dx=-e^{\frac{1}{x}}+C.$
- $\int e^{5x-3}\ dx=\frac{1}{5}\cdot\int 5\cdot e^{5x-3}\ dx=\frac{1}{5}\cdot e^{5x-3}+C.$

Exercici 7. Resol les següents integrals $\int 4^{x^2+3}\cdot x$ dx, $\int\ 8^{5x-3}$dx.

- $\int 4^{x^2+3}\cdot x\ dx=\ \frac{1}{2}\cdot\int 4^{x^2+3}\cdot 2x\ dx=\frac{4^{x^2+3}}{2\cdot\ln 4}+C.$
- $\int\ 8^{5x-3}dx=\frac{8^{5x-3}}{5\cdot\ln 8}+C.$

Exercici 8. Resol les següents integrals $\int$ sinx$^2\cdot$x dx, $\int\ \frac{\sin(\ln x)}{x}$dx, $\int$ sin(cosx)$\cdot$sinx dx.

- $\int$ sinx$^2\cdot$x dx=$\frac{1}{2}\cdot\int$ sinx$^2\cdot$2x dx=-$\frac{1}{2}\cdot$cosx^2+C.
- $\int\ \frac{\sin(\ln x)}{x}$dx=-cos(lnx)+C.
- $\int$ sin(cosx)$\cdot$sinx dx=-cos(cosx)+C.

Exercici 9. Resol les següents integrals $\int$ cos(e^{3x})·e^{3x} dx, $\int$cos(8x-9) dx, $\int$ cos(sinx)·cosx dx.

- $\int$ cos(e^{3x})·e^{3x} dx=$\frac{-1}{3}$·sine^{3x}+C.
- $\int$cos(8x-9) dx=$\frac{1}{8}$·sin(8x-9)+C.
- $\int$ cos(sinx)·cosx dx=cos(sinx)+C.

Exercici 10. Resol les següents integrals $\int \frac{x}{\sqrt{2-x^4}}$·dx, $\int \frac{\frac{1}{x}}{\sqrt{1-\ln^2 x}}$·dx.

- $\int \frac{x}{\sqrt{2-x^4}}$·dx= $\frac{1}{\sqrt{2}}$·$\int \frac{x}{\sqrt{1-\frac{1}{2}(x^2)^2}}$·dx=$\frac{1}{\sqrt{2}}$·$\frac{\sqrt{2}}{2}$ $\int \frac{\frac{2}{\sqrt{2}}x}{\sqrt{1-(\frac{1}{\sqrt{2}}x^2)^2}}$·dx=$\frac{1}{2}$·arcsin($\frac{1}{\sqrt{2}}x^2$)+C.
- $\int \frac{\frac{1}{x}}{\sqrt{1-\ln^2 x}}$·dx.=arcsin(lnx)+C.

Exercici 11. Resol les següents integrals $\int \frac{5}{8+7x^2}$·dx, $\int \frac{x}{1+x^4}$·dx

- $\int \frac{5}{8+7x^2}$·dx=5$\int \frac{1}{8\cdot(1+\frac{7x^2}{8})}$dx=5·$\sqrt{\frac{7}{8}}$·$\frac{1}{8}$·$\int \frac{\sqrt{\frac{7}{8}}}{(1+\frac{\sqrt{7}x}{\sqrt{8}})^2}$dx=5·$\sqrt{\frac{7}{8}}$·$\frac{1}{8}$·arctag($\sqrt{\frac{7}{8}}$·x)+C.
- $\int \frac{x}{1+x^4}$·dx=$\frac{1}{2}$·arctagx^2+C.

Exercicis per resoldre

1. Calcula les següents integrals:

   - $\int(x^2 + 8x - 9)$ dx
   - $\int(4x^3 + 8x^2-2x+4)$ dx
   - $\int(\frac{-5}{x^2}+4x-2)$ dx
   - $\int(7\sqrt{x} + 3x^4+2x^3 - 7\sqrt[5]{x})$ dx
   - $\int(2x^3 + \frac{1}{\sqrt[4]{x^3}})$ dx
   - $\int(\frac{3}{x^5} - \frac{2}{x^4} - 3\sqrt[5]{x^4})$ dx
   - $\int(x-1)^7$ dx
   - $\int(5x-9)^6$ dx

- $\int(3x^2\text{-}9)^4\cdot x\ dx$
- $\int(\sin x\text{-}4)^5\cdot\cos x\ dx$
- $\int(\ln x\text{-}1)^4\ \cdot\frac{1}{x}dx$
- $\int\ \sqrt[3]{(9x\text{-}4)^5}\ dx$
- $\int\ \sqrt{5e^x\text{-}4}\cdot e^x dx$
- $\int\frac{4x}{x^2+2}\ dx$
- $\int\frac{\sin x}{4+3\cos x}\ dx$
- $\int\frac{\sin x+\cos x}{\sin x\text{-}\cos x}\ dx$
- $\int\frac{1}{x\ln x}\ dx$
- $\int e^{3x-9}\ dx$
- $\int e^{x^2+9}\cdot x\ dx$
- $\int\frac{e^{\sqrt{x}}}{\sqrt{x}}\ dx$
- $\int\sin(2x\text{-}4)\ dx$
- $\int\sin(3x^2\text{-}4)\ x\ dx$
- $\int\sin(e^x\text{-}3)\ e^x\ dx$
- $\int\frac{\cos(\ln x)}{x}\ dx$
- $\int\frac{\cos(\sqrt{x})}{\sqrt{x}}\ dx$
- $\int\frac{x}{\cos^2 x^2}\ dx$
- $\int\frac{1}{\sqrt{x}\cos^2(\sqrt{x})}\ dx$
- $\int\frac{2}{\sqrt{2\text{-}x^2}}\ dx$
- $\int\frac{8x}{\sqrt{5\text{-}x^4}}\ dx$
- $\int\frac{1}{x\cdot\sqrt{1\text{-}(\ln x)^2}}\ dx$
- $\int\frac{8}{5+3x^2}\ dx$
- $\int\frac{2x}{6+7x^4}\ dx$
- $\int\frac{8}{5+3x^2}\ dx$
- $\int\frac{1}{(1+x)\cdot\sqrt{x}}\ dx$
- $\int\frac{x^3}{x^2+5}\ dx$
- $\int\frac{x+1}{x^2-1}\ dx$

- $\int 6^{\text{2x-5}}\ \text{dx}$
- $\int \sqrt{\text{1-5x}}\ \text{dx}$
- $\int (\text{3x}+\frac{1}{x^2})\ \text{dx}$
- $\int \frac{\text{dx}}{(x-1)^{12}}$
- $\int \sqrt{\text{2-x}^2}\cdot\text{x dx}$
- $\int \frac{1}{\sqrt{\text{1-x}}}\ \text{dx}$
- $\int \frac{\text{x}}{\sqrt{\text{1-x}^2}}\ \text{dx}$
- $\int \frac{\text{6x+3}}{\sqrt{\text{x+x}^2}}\ \text{dx}$
- $\int \frac{1}{x\cdot(\text{lnx})^5}\ \text{dx}$
- $\int \frac{(\text{arcsinx})^4}{\sqrt{\text{1-x}^2}}\ \text{dx}$
- $\int \frac{\text{cosx}}{\sqrt{\text{sinx}}}\ \text{dx}$
- $\int \frac{1}{\sqrt{\text{1-x}^2}\cdot(\text{arcosx})^2}\ \text{dx}$
- $\int \frac{\text{cosx}}{\text{2+2sinx}}\ \text{dx}$
- $\int (\text{x+tagx})\ \text{dx}$
- $\int \frac{e^{\text{arcyagx}}}{1+x^2}\ \text{dx}$
- $\int \text{e}^{\text{x}}+\text{e}^{\text{x}^2}x+\frac{1}{e^{\text{4x}}}-x^2\cdot\text{e}^{\text{x}^3}\ \text{dx}$
- $\int (\text{sin3x+cos5x})\ \text{dx}$
- $\int \frac{dx}{\cos^2 6x}$
- $\int 6^{\text{x}^2\text{+8x}}\cdot(\text{x+4})\ \text{dx}$
- $\int \frac{4^x}{5^x}\ \text{dx}$
- $\int \text{x}^2\ \text{sinx}^3\ \text{dx}$
- $\int \frac{\sin\frac{1}{x}}{\frac{1}{x^2}}\ \text{dx}$
- $\int \frac{\sin\sqrt{x}+\cos\sqrt{x}}{\sqrt{x}}\ \text{dx}$
- $\int \frac{\sin(\ln x)}{x}\ \text{dx}$
- $\int \frac{\text{e}^x}{1+e^{2x}}\ \text{dx}$
- $\int \frac{\text{x+1}}{x^2+1}\ \text{dx}$
- $\int \frac{\text{12x+3}}{12x^2+3}\ \text{dx}$
- $\int \frac{\text{1+cosx+sin}^2\text{x+sin}^3\text{x}}{\sin^2 x}\ \text{dx}$

- $\int \sin^2 x$ dx
- $\int \cos^2 x$ dx
- $\int \frac{2}{\sqrt{1\text{-}5x^2}}$ dx
- $\int \frac{1\text{-}x}{\sqrt{1\text{-}x^2}}$ dx
- $\int \frac{e^x}{\sqrt{1\text{-}e^{2x}}}$ dx
- $\int \frac{\text{-}2}{\sqrt{1\text{-}x^2}}$ dx
- $\int 5^{3x}$ dx
- $\int \frac{\ln(x+1)}{x+1}$ dx
- $\int \frac{6}{3x^2+7}$ dx
- $\int \sin(x^2+1)\cdot x$ dx
- $\int \sqrt{x^3+1}\cdot x^2$ dx

2. Troba una funció que verifiqui f'(x)=5x-3 i tal que f(4)=2.

3. Troba un funció tal que la seva derivada sigui x^2+5x-2 i que en el punt 3 valgui 5.

4. Troba una funció tal que f'(x)=$\frac{x}{x^2+1}$ i que f(2)=4.

5. Troba una funció tal que f'(x)=$\frac{1}{3x+1}$ i que f(0)=8.