

Solucions examen 5

1.- Siguin $\sin x = \frac{3}{4}$ amb $90^\circ < x < 180^\circ$ i $\text{tag} y = 5$ $180^\circ < y < 270^\circ$. Calcula: $\sin(180-(x))$, $\cos(x+y)$, $\text{tag}(x-y)$, $\sin(2x)$

En efecte,

$$\left(\frac{3}{4}\right)^2 + \cos^2 x = 1$$

$$\frac{9}{16} + \cos^2 x = 1$$

$$\cos^2 x = 1 - \frac{9}{16} = \frac{7}{16}$$

$$\cos x = -\sqrt{\frac{7}{16}} = -\frac{\sqrt{7}}{4}$$

$$\begin{cases} \frac{\sin y}{\cos y} = 5 \\ \sin^2 y + \cos^2 y = 1 \end{cases} \rightarrow \begin{cases} \sin y = 5 \cos y \\ \sin^2 y + \cos^2 y = 1 \end{cases}$$

$$25 \cos^2 y + \sin^2 y = 1$$

$$26 \cos^2 y = 1$$

$$\cos y = \sqrt{\frac{1}{26}} = \frac{1}{\sqrt{26}}$$

$$\sin y = \frac{5}{\sqrt{26}}$$

Aleshores,

$$\sin(180-(x)) = \sin x = \frac{3}{4}$$

$$\cos(x+y) = \cos x \cdot \cos y - \sin x \cdot \sin y = -\frac{\sqrt{7}}{4} \cdot \left(\frac{1}{\sqrt{26}}\right) - \frac{3}{4} \cdot \left(\frac{5}{\sqrt{26}}\right) = \frac{-\sqrt{7}-15}{4\sqrt{26}}$$

$$\text{tag}(x-y) = \frac{\sin(x-y)}{\cos(x-y)} = \frac{\sin x \cdot \cos y - \cos x \cdot \sin y}{\cos x \cdot \cos y + \sin x \cdot \sin y} = \frac{\frac{3}{4} \cdot \frac{1}{\sqrt{26}} - \left(-\frac{\sqrt{7}}{4}\right) \cdot \frac{5}{\sqrt{26}}}{-\frac{\sqrt{7}}{4} \cdot \left(\frac{1}{\sqrt{26}}\right) + \frac{3}{4} \cdot \left(\frac{5}{\sqrt{26}}\right)} = \frac{\frac{3+5\sqrt{7}}{4\sqrt{26}}}{\frac{-\sqrt{7}+15}{4\sqrt{26}}} = \frac{3+5\sqrt{7}}{-\sqrt{7}+15}$$

$$\sin 2x = 2 \sin x \cos x = 2 \cdot \frac{3}{4} \cdot \left(-\frac{\sqrt{7}}{4}\right) = \frac{-6\sqrt{7}}{16} = \frac{-3\sqrt{7}}{8}$$

2.- Resol en triangle que mesura $a=5\text{cm}$, $A=35^\circ$, $B=50^\circ$. Quina és l' àrea d' aquest triangle?

En efecte,

$$C = 180 - 35 - 50 = 95^\circ$$

$$\frac{5}{\sin 35} = \frac{b}{\sin 50} \rightarrow b = \frac{5 \sin 50}{\sin 35} = 6,67cm$$

$$\frac{5}{\sin 35} = \frac{c}{\sin 95} \rightarrow c = \frac{3 \sin 95}{\sin 35} = 5,21cm$$

D' altra banda l' àrea val,

$$A = \frac{ab \sin C}{2} = \frac{5 \cdot 6,67 \cdot \sin 95}{2} = 16,61cm^2$$

3.- Resol les equacions:

a) $\sin x = -0,9$

b) $\cos 3x = 0,1$

En efecte,

$$\sin x = -0,9$$

$$64,15^\circ$$

$$x = (180 + 64,16) + 360k = 244,16 + 360K$$

$$x = (360 - 64,15) + 360k = 295,84 + 360K$$

$$\cos 3x = 0,1$$

$$3x = 84,26 + 360K \rightarrow x = \frac{84,26 + 360K}{3}$$

$$3x = (360 - 84,26) + 360K \rightarrow x = \frac{275,74 + 360K}{3}$$

4.- Demuestra la identitat

$$\operatorname{tag} a - \operatorname{tag} b = \frac{\sin(a-b)}{\cos a \cos b}$$

En efecte,

$$\bullet \frac{\sin(a-b)}{\cos a \cos b} = \frac{\sin a \cos b - \sin b \cos a}{\cos a \cos b} = \frac{\sin a \cos b}{\cos a \cos b} - \frac{\sin b \cos a}{\cos a \cos b} = \frac{\sin a}{\cos a} - \frac{\sin b}{\cos b} = \tan a - \tan b$$

5.- Dos pobles estan separats per 500 m. Un avió està volant entre aquests dos pobles de tal manera que el poble A el veu segons un angle de 30° i des del B segons un angle de 80° . A quina alçada vola l'avió?

En efecte,

$$\frac{500}{\sin 70} = \frac{b}{\sin 30} \rightarrow b = \frac{500 \sin 30}{\sin 70} = 266$$

$$\sin 80 = \frac{h}{266} \rightarrow h = 266 \sin 80 = 262m$$