Solucions examen 6

1.- Siguin sinx=$\frac{-2}{5}$ essent 180^o<x<270^o i tagy=-2 essent 90^o<y<270^o. Calcula: sin(180+(x)), sin(x-y), tag(x+y), sin(2x) i cos(y/2).

En efecte,

$$(\frac{-2}{5})^2+\cos^2 x=1$$

$$\frac{4}{25}+\cos^2 x=1$$

$$\cos^2 x=1-\frac{4}{25}=\frac{21}{25}$$

$$\cos x=-\sqrt{\frac{21}{25}}=-\frac{\sqrt{21}}{5}$$

$$\left\{\begin{array}{c}\frac{\sin y}{\cos y}=-2\\ \sin^2 y+\cos^2 y=1\end{array}\right. \rightarrow \left\{\begin{array}{c}\sin y=-2\cos y\\ \sin^2 y+\cos^2 y=1\end{array}\right.$$

$$4\cos^2 y+\sin^2 y=1$$

$$5\cos^2 y=1$$

$$\begin{aligned}\cos y &= \sqrt{\frac{1}{5}}=\frac{-1}{\sqrt{5}}\\ \sin y &= \frac{2}{\sqrt{5}}\end{aligned}$$

Aleshores,

sin(180+x)=-sinx=$\frac{2}{5}$

$\sin(x-y)=\sin x$·cosy-cosx·siny=-$\frac{2}{5}\cdot(\frac{-1}{\sqrt{5}})-(-\frac{\sqrt{21}}{5})\cdot\frac{2}{\sqrt{5}}=\frac{2+2\sqrt{21}}{5\sqrt{5}}$

tag(x+y)=$\frac{\sin(x+y)}{\cos(x+y)}=\frac{\sin x\cdot \text{cosy}+\text{cosx}\cdot\text{siny}}{cosx\cdot cosy-sinx\cdot siny}=\frac{-\frac{2}{5}\cdot(\frac{-1}{\sqrt{5}})+(-\frac{\sqrt{21}}{5})\cdot\frac{2}{\sqrt{5}}=\frac{2+2\sqrt{21}}{5\sqrt{5}}}{-\frac{\sqrt{21}}{5}\cdot(\frac{-1}{\sqrt{5}})-(-\frac{2}{5})\cdot(\frac{2}{\sqrt{5}})}=\frac{\frac{2-2\sqrt{21}}{5\sqrt{5}}}{\frac{\sqrt{21}+4}{5\sqrt{5}}}=\frac{2-2\sqrt{21}}{\sqrt{21}+4}$

sin2x=2sinxcosx=2$(\frac{-2}{5})(-\frac{\sqrt{21}}{5})=\frac{4\sqrt{21}}{25}$

$\cos(\frac{y}{2})=\sqrt{\frac{1+\cos y}{2}}=\sqrt{\frac{1+(\frac{-1}{\sqrt{5}})}{2}}=\sqrt{\frac{\sqrt{5}-1}{2\sqrt{5}}}$

2.- La base d' un triangle isòsceles mesura 10 cm i l' angle oposat 50^o. Calcula l' àrea del triangle.

$$
\begin{aligned}
tag25^o &= \frac{catet \text{ oposat}}{h} \rightarrow h = \frac{5}{tag25} = 10,7 \\
A &= \frac{10 \cdot \frac{5}{tag25}}{2} = \frac{25}{tag25} = 53,6
\end{aligned}
$$

3.- Resol la següent equació trigonomètrica cos^{2}x-3sin^{2}x=0

$$
\begin{aligned}
1 - \sin^2 x - 3\sin^2 x &= 0 \\
-4\sin^2 x &= -1 \\
\sin^2 x &= \frac{1}{4} \\
\sin x &= \pm\sqrt{\frac{1}{4}} = \pm\frac{1}{2}
\end{aligned}
$$

$$
\sin x = \frac{1}{2} \rightarrow \begin{cases} x = 30 + 360K \\ x = 150 + 360k \end{cases}
$$

$$
\sin x = -\frac{1}{2} \rightarrow \begin{cases} x = 210 + 360K \\ x = 330 + 360k \end{cases}
$$

4.- Comprova la següent identitat

$$
\frac{\sin x + tagx}{\cot agx + \cos ecx} = \sin x tagx
$$

$$
\begin{aligned}
\frac{\sin x + tagx}{\cot agx + \cos ecx} &= \frac{\sin x + \frac{\sin x}{\cos x}}{\frac{\cos x}{\sin x} + \frac{1}{\sin x}} = \frac{\frac{\sin x \cos x + \sin x}{\cos x}}{\frac{\cos x + 1}{\sin x}} \\
&= \frac{\sin x}{\cos x}\frac{\sin x(\cos x + 1)}{\cos x + 1} = \frac{\sin x}{\cos x}\cdot \text{sinx} = \sin x tagx
\end{aligned}
$$

5.- Resol el triangle a=7cm, A=15^o i B=30^o

$$
C = 180 - 15 - 30 = 135
$$

$$
\begin{aligned}
\frac{7}{\sin 15} &= \frac{b}{\sin 30} \rightarrow b = \frac{7\sin 30}{\sin 15} = 13,52 \\
\frac{7}{\sin 15} &= \frac{c}{\sin 135} \rightarrow c = \frac{7\sin 135}{\sin 15} = 19,1
\end{aligned}
$$

6.- Resol

$$\frac{2}{x-1}+\frac{4}{x-2}=5$$
$$-x^2+8x+9<0$$

En efecte

$$\frac{2}{x-1}+\frac{4}{x-2}=5$$
$$\frac{2(x-2)}{(x-1)(x-2)}+\frac{4(x-1)}{(x-1)(x-2)}=\frac{5(x-1)(x-2)}{(x-1)(x-2)}$$
$$2x-4+4x-4=5x^2-15x+10$$
$$5x^2-21x+18=0$$
$$x=\frac{21\pm\sqrt{441-360}}{10}=\frac{21\pm 9}{10}=\left\{\begin{array}{c} x=3 \\ x=\frac{12}{10}=\frac{6}{5}\end{array}\right.$$

$$-x^2+8x+9<0$$
$$x=\frac{-8\pm\sqrt{64+36}}{-2}=\frac{-8\pm 10}{-2}=\left\{\begin{array}{c} x=-1 \\ x=9\end{array}\right.$$

Solució:

$$(-\infty,-1)\cup(9,+\infty)$$