Solucions examen 4

1.- *Donades les matrius*

$$A=\begin{pmatrix}3&2&2\\2&2&4\\1&-1&3\end{pmatrix} \text{ i } B=\begin{pmatrix}-2&1&3\\0&2&1\\0&4&5\end{pmatrix}$$

Calculeu:

a) 3A-5B

b) A•B-^{t}B•A

c) detA i detB

d) B^3

En efecte:

a) $3A-5B=3\begin{pmatrix}3&2&2\\2&2&4\\1&-1&3\end{pmatrix}-5\begin{pmatrix}-2&1&3\\0&2&1\\0&4&5\end{pmatrix}=\begin{pmatrix}19&1&-9\\6&-4&7\\3&-23&-16\end{pmatrix}$

b) $A\bullet B=\begin{pmatrix}3&2&2\\2&2&4\\1&-1&3\end{pmatrix}\cdot\begin{pmatrix}-2&1&3\\0&2&1\\0&4&5\end{pmatrix}=\begin{pmatrix}-6&15&21\\-4&22&28\\-2&11&17\end{pmatrix}$

$^t\text{B}\cdot\text{A}=\begin{pmatrix}-2&0&0\\1&2&4\\3&1&5\end{pmatrix}\cdot\begin{pmatrix}3&2&2\\2&2&4\\1&-1&3\end{pmatrix}=\begin{pmatrix}-6&-4&-4\\11&2&22\\16&0&25\end{pmatrix}$

$A\bullet B-{}^t\text{B}\cdot\text{A}=\begin{pmatrix}-6&15&21\\-4&22&28\\-2&11&17\end{pmatrix}-\begin{pmatrix}-6&-4&-4\\11&2&22\\16&0&25\end{pmatrix}=\begin{pmatrix}0&19&25\\-15&20&6\\-18&11&-8\end{pmatrix}$

c)

$$\text{detA}=\begin{vmatrix}3&2&2\\2&2&4\\1&-1&3\end{vmatrix}=3\cdot(10)-2\cdot(8)+1\cdot4=18$$

$$\text{detB}=\begin{vmatrix}-2&1&3\\0&2&1\\0&4&5\end{vmatrix}=-2\cdot6=-12$$

d)

$$B^3 = \begin{pmatrix} -2 & 1 & 3 \\ 0 & 2 & 1 \\ 0 & 4 & 5 \end{pmatrix}^3 = \begin{pmatrix} -2 & 1 & 3 \\ 0 & 2 & 1 \\ 0 & 4 & 5 \end{pmatrix} \begin{pmatrix} -2 & 1 & 3 \\ 0 & 2 & 1 \\ 0 & 4 & 5 \end{pmatrix} \begin{pmatrix} -2 & 1 & 3 \\ 0 & 2 & 1 \\ 0 & 4 & 5 \end{pmatrix} =$$

$$\begin{pmatrix} -4 & 12 & 10 \\ 0 & 8 & 7 \\ 0 & 28 & 29 \end{pmatrix} \begin{pmatrix} -2 & 1 & 3 \\ 0 & 2 & 1 \\ 0 & 4 & 5 \end{pmatrix} = \begin{pmatrix} 8 & 60 & 50 \\ 0 & 44 & 43 \\ 0 & 172 & 173 \end{pmatrix}$$

2.- Estudia i resol el sistema

$$\begin{cases} 2x + y + 5z = 3 \\ 3x + 4y + 2z = 2 \\ 5x + y + 2z = 5 \end{cases}$$

A= $\begin{pmatrix} 2 & 1 & 5 \\ 3 & 4 & 2 \\ 5 & 1 & 2 \end{pmatrix}$ i B= $\begin{pmatrix} 2 & 1 & 5 & 3 \\ 3 & 4 & 2 & 2 \\ 5 & 1 & 2 & 5 \end{pmatrix}$

RangA?

detA=$2\cdot 6 - 3\cdot(-3) + 5\cdot(-18) = -69$. Aleshores $\begin{cases} RangA = 3 \\ RangB = 3 \\ n = 3 \end{cases}$

Les solucions seran

$$\text{x}=\frac{\begin{vmatrix} 3 & 1 & 5 \\ 2 & 4 & 2 \\ 5 & 1 & 2 \end{vmatrix}}{\begin{vmatrix} 2 & 1 & 5 \\ 3 & 4 & 2 \\ 5 & 1 & 2 \end{vmatrix}} = \frac{-66}{-69} = \frac{22}{23} \qquad y=\frac{\begin{vmatrix} 2 & 3 & 5 \\ 3 & 2 & 2 \\ 5 & 5 & 2 \end{vmatrix}}{\begin{vmatrix} 2 & 1 & 5 \\ 3 & 4 & 2 \\ 5 & 1 & 2 \end{vmatrix}} = \frac{25}{-69}$$

$$z=\frac{\begin{vmatrix} 2 & 1 & 3 \\ 3 & 4 & 2 \\ 5 & 1 & 5 \end{vmatrix}}{\begin{vmatrix} 2 & 1 & 5 \\ 3 & 4 & 2 \\ 5 & 1 & 2 \end{vmatrix}} = \frac{20}{69}$$

3.- *Donada la matriu* $B=$ $\begin{pmatrix} -2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 4 & 5 \end{pmatrix}$

a) Troba B^{-1}

b) Resol el sistema

$$\begin{cases} -2x + y = 3 \\ x + 2y + z = 1 \\ 4y + 5z = 4 \end{cases}$$

En efecte,

i) detB=-2·6-1·5=-17

ii) ${}^{t}\mathrm{B}=\begin{pmatrix} -2 & 1 & 0 \\ 1 & 2 & 4 \\ 0 & 1 & 5 \end{pmatrix}$

iii) ${}^{adj}({}^{t}\mathrm{B})=\begin{pmatrix} 6 & -5 & 1 \\ -5 & -10 & 2 \\ 4 & 8 & -5 \end{pmatrix}$

$$\mathrm{B}^{-1} = \begin{pmatrix} \frac{-6}{17} & \frac{5}{17} & \frac{-1}{17} \\ \frac{5}{17} & \frac{10}{17} & \frac{-2}{17} \\ \frac{-4}{17} & \frac{-8}{17} & \frac{5}{17} \end{pmatrix}$$

Finalment les solucions del sistema seran, $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \frac{-6}{17} & \frac{5}{17} & \frac{-1}{17} \\ \frac{5}{17} & \frac{10}{17} & \frac{-2}{17} \\ \frac{-4}{17} & \frac{-8}{17} & \frac{5}{17} \end{pmatrix} \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$

4.- Calcula $\begin{vmatrix} 2 & 1 & 3 & 1 \\ 3 & 1 & 4 & 3 \\ 1 & 0 & 0 & 0 \\ 4 & 2 & 1 & 2 \end{vmatrix} = 1 \begin{vmatrix} 1 & 3 & 1 \\ 1 & 4 & 3 \\ 2 & 1 & 2 \end{vmatrix} = 5 - 1\cdot 5 + 2\cdot 5 = 10$