Solucions examen 4

Estudia i resol el sistema

$$\begin{cases} 2x+3y+4z=5 \\ x-y+8z=1 \\ x+3y+5z=2 \end{cases}$$

A= $\begin{pmatrix} 2 & 3 & 4 \\ 1 & -1 & 8 \\ 1 & 3 & 5 \end{pmatrix}$ i B= $\begin{pmatrix} 2 & 3 & 4 & 5 \\ 1 & -1 & 8 & 1 \\ 1 & 3 & 5 & 2 \end{pmatrix}$

RangA?

detA=$2\cdot(-29)-1\cdot(3)+1\cdot(28)=-33$. Aleshores $\begin{cases} RangA=3 \\ RangB=3 \\ n=3 \end{cases}$

Les solucions seran

$$x=\frac{\begin{vmatrix} 5 & 3 & 4 \\ 1 & -1 & 8 \\ 2 & 3 & 5 \end{vmatrix}}{\begin{vmatrix} 2 & 3 & 4 \\ 1 & -1 & 8 \\ 1 & 3 & 5 \end{vmatrix}}=\frac{-92}{-33} \qquad y=\frac{\begin{vmatrix} 2 & 5 & 4 \\ 1 & 1 & 8 \\ 1 & 2 & 5 \end{vmatrix}}{\begin{vmatrix} 2 & 3 & 4 \\ 1 & -1 & 8 \\ 1 & 3 & 5 \end{vmatrix}}=\frac{-3}{-33}$$

$$z=\frac{\begin{vmatrix} 2 & 3 & 5 \\ 1 & -1 & 1 \\ 1 & 3 & 2 \end{vmatrix}}{\begin{vmatrix} 2 & 3 & 4 \\ 1 & -1 & 8 \\ 1 & 3 & 5 \end{vmatrix}}=\frac{7}{-33}$$

$$\begin{cases} x-y+z=4 \\ x+2y+4z=3 \\ 2x+y+5z=7 \end{cases}$$

A= $\begin{pmatrix} 1 & -1 & 1 \\ 1 & 2 & 4 \\ 2 & 1 & 5 \end{pmatrix}$ i B= $\begin{pmatrix} 1 & -1 & 1 & 4 \\ 1 & 2 & 4 & 3 \\ 2 & 1 & 5 & 7 \end{pmatrix}$

RangA?

detA=0.

$$\begin{vmatrix} 1 & -1 \\ 1 & 2 \end{vmatrix} = 3$$

Rang B?

$$\begin{vmatrix} 1 & -1 & 4 \\ 1 & 2 & 3 \\ 2 & 1 & 7 \end{vmatrix} = 0, \quad \begin{vmatrix} 1 & 4 & 1 \\ 1 & 3 & 4 \\ 2 & 7 & 5 \end{vmatrix} = 0 \text{ i } \begin{vmatrix} -1 & 1 & 4 \\ 2 & 4 & 3 \\ 1 & 5 & 7 \end{vmatrix} = 0$$

Aleshores $\begin{cases} RangA = 2 \\ RangB = 2 \\ n = 3 \end{cases}$

Per tant, estem davant un sistema compatible indeterminat amb un grau de llibertat (z).

Les solucions seran

$$\begin{cases} x - y = 4 - z \\ x + 2y = 3 - 4z \end{cases}$$

$$\text{x}=\frac{\begin{vmatrix} 4-z & -1 \\ 3-4z & 2 \end{vmatrix}}{\begin{vmatrix} 1 & -1 \\ 1 & 2 \end{vmatrix}} = \frac{11-6z}{3} \qquad y=\frac{\begin{vmatrix} 1 & 4-z \\ 1 & 3-4z \end{vmatrix}}{\begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix}} = \frac{-1-3z}{3}$$

$$\begin{cases} x + y + z = 3 \\ x - y + 4z = 1 \\ 2x + 5z = 7 \end{cases}$$

$$\text{A}= \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 4 \\ 2 & 0 & 5 \end{pmatrix} \text{ i B}= \begin{pmatrix} 1 & 1 & 1 & 3 \\ 1 & -1 & 4 & 1 \\ 2 & 0 & 5 & 7 \end{pmatrix}$$

RangA?

detA=0.

$$\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = -2$$

Rang B?

$$\begin{vmatrix} 1 & 1 & 3 \\ 1 & -1 & 1 \\ 2 & 0 & 7 \end{vmatrix} = -4 - 4 + 2 \cdot 7 = 0,$$

Aleshores $\begin{cases} RangA = 2 \\ RangB = 3 \\ n = 3 \end{cases}$

Per tant, estem davant un sistema incompatible