Exemple 1. *Simplifica les següents expressions*

(i)
$$\frac{x^5-18x^3-4x^2+57x-36}{x^4-13x^3+60x^2-112x+64}$$

(ii)
$$\frac{x^4+8x^3+14x^2-8x-15}{x^3+9x^2+15x-25}$$

Resolució. (i)
$$\frac{x^5-18x^3-4x^2+57x-36}{x^4-13x^3+60x^2-112x+64}$$
Si descomposem els polinomis $x^5-18x^3-4x^2+57x-36$ i $x^4-13x^3+60x^2-112x+64$ per Ruffini resulta que

$$x^5-18x^3-4x^2+57x-36=(x-1)^2(x+3)^2(x-4)$$
$$x^4-13x^3+60x^2-112x+64=(x-1)(x-4)^3$$

D' aquesta manera,
$$\frac{x^5-18x^3-4x^2+57x-36}{x^4-13x^3+60x^2-112x+64}=\frac{(x-1)^2(x+3)^2(x-4)}{(x-1)(x-4)^3}=\frac{(x-1)(x+3)^2}{(x-4)^2}$$

(ii)
$$\frac{x^4+8x^3+14x^2-8x-15}{x^3+9x^2+15x-25}$$
Descomposem els polinomis $x^4+8x^3+14x^2-8x-15$ i $x^3+9x^2+15x-25$ per Ruffini i resulta que

$$x^4+8x^3+14x^2-8x-15=(x-1)(x+1)(x+3)(x+5)$$
$$x^3+9x^2+15x-25=(x-1)(x+5)^2$$

D' aquesta manera,
$$\frac{x^4+8x^3+14x^2-8x-15}{x^3+9x^2+15x-25}=\frac{(x-1)(x+1)(x+3)(x+5)}{(x-1)(x+5)^2}=\frac{(x+1)(x+3)}{(x+5)}$$

Exemple 2. *Calcula les següents sumes i restes*

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(i)

$$\frac{2x+3}{x^3-3x+2}+\frac{x+5}{x^3-2x^2-5x+6}$$

(ii)

$$\frac{2x+4}{x^2-1}+\frac{3x-2}{x+1}-\frac{4x-5}{x-1}$$

Resolució.(i)

$$\frac{2x+3}{x^3-3x+2}+\frac{x+5}{x^3-2x^2-5x+6}$$

Calculem, primerament, el mcm dels polinomis x^3-3x+2 i x^3-2x^2-5x+6. Descomposem, doncs, per Ruffini aquests dos polinomis i el resultat és

$$x^3-3x+2=(x-1)^2(x+2)$$

$$x^3-2x^2-5x+6=(x-1)(x+2)(x-3)$$

D' aquesta manera,

$$mcm(x^3-3x+2,x^3-2x^2-5x+6)=(x-1)^2(x+2)(x-3)$$

Aleshores,

$$\frac{2x+3}{x^3-3x+2}+\frac{x+5}{x^3-2x^2-5x+6}=\frac{(2x+3)(x-3)+(x+5)(x-1)}{(x-1)^2(x+2)(x-3)}=$$

$$\frac{3x^2-3x-2}{(x-1)^2(x+2)(x-3)}$$

(ii)

$$\frac{2x+4}{x^2-1}+\frac{3x-2}{x+1}-\frac{4x-5}{x-1}$$

Calculem, primerament, el mcm dels polinomis x^2-1, $x+1$ i $x-1$. Descomposem, doncs, aquests tres polinomis i el resultat és

$$x^2-1=(x-1)(x+1)$$

$$x-1=x-1$$

$$x+1=x+1$$

D' aquesta manera,

$$mcm(x^2-1, x+1, x-1) = (x-1)(x+1)$$

Aleshores,

$$\frac{2x+4}{x^2-1} + \frac{3x-2}{x+1} - \frac{4x-5}{x-1} = \frac{2x+4+(3x-2)(x-1)-(4x-5)(x+1)}{(x-1)(x+1)}$$

$$\frac{-x^2-2x+11}{(x-1)(x+1)}$$

Exemple 3. *Calcula les següents multiplicacions i divisions*

(i)
$$\frac{x^2+3x-2}{x-1} \cdot \frac{x^2+4x-3}{x+2}$$

(ii)
$$\frac{x+3}{x-1} \cdot \frac{x^2-1}{x^2-9}$$

(iii)
$$\frac{x^2+2x-3}{x^2+2x-8} \cdot \frac{x-2}{x-1}$$

(iv)
$$\frac{x^2+3x-2}{x-1} : \frac{x^2+4x-3}{x+2}$$

(v)
$$\frac{x+1}{x-2} : \frac{x+3}{x^2-4}$$

R*esolució*.

(i)
$$\frac{x^2+3x-2}{x-1} \cdot \frac{x^2+4x-3}{x+2} = \frac{(x^2+3x-2)\cdot(x^2+4x-3)}{(x-1)\cdot(x+2)}$$

$$= \frac{x^4+7x^3+7x^2-17x+6}{x^2+x-2}$$

(ii)

$$\frac{x+3}{x-1}\cdot\frac{x^2-1}{x^2-9}=\frac{(x+3)\cdot(x^2-1)}{(x-1)\cdot(x^2-9)}=\frac{(x+3)\cdot(x+1)\cdot(x-1)}{(x-1)\cdot(x+3)\cdot(x-3)}=\frac{x+1}{x-3}$$

(iii)

$$\frac{x^2+2x-3}{x^2+2x-8}\cdot\frac{x-2}{x-1}=\frac{(x^2+2x-3)(x-2)}{(x^2+2x-8)(x-1)}=\frac{(x+3)\cdot(x-1)\cdot(x-2)}{(x-2)\cdot(x+4)\cdot(x-1)}$$

$$=\frac{x+3}{x+4}$$

(iv)

$$\frac{x^2-3x-2}{x-1}:\frac{x^2+4x-3}{x+2}=\frac{(x^2-3x-2)\cdot(x+2)}{(x-1)\cdot(x^2+2x-3)}=\frac{(x-1)\cdot(x-2)\cdot(x+2)}{(x-1)\cdot(x-1)(x+3)}$$

$$=\frac{(x-2)\cdot(x+2)}{(x-1)(x+3)}=\frac{x^2-4}{x^2+2x-3}$$

(v)

$$\frac{x+1}{x-2}:\frac{x+3}{x^2-4}=\frac{(x+1)\cdot(x^2-4)}{(x-2)\cdot(x+3)}=\frac{(x+1)\cdot(x+2)\cdot(x-2)}{(x+2)\cdot(x+3)}$$

$$=\frac{(x+1)\cdot(x+2)\cdot(x-2)}{(x+2)\cdot(x+3)}=\frac{(x+1)\cdot(x-2)}{(x+2)\cdot(x+3)}=\frac{x^2-x-2}{x^2+5x+6}$$

Exemple 4. *Efectua les següents operacions*

(i)

$$\frac{\frac{x+1}{x-2}-\frac{x-1}{x+3}}{\frac{x+2}{x+3}-\frac{x-1}{x+2}}$$

(ii)

$$\frac{1-\frac{1}{x}}{1+\frac{1}{x}}$$

(iii)

$$\frac{\frac{x+3}{x+1}-\frac{x+4}{x-1}}{\frac{x+1}{x-1}+\frac{x+3}{x+1}}$$

R*esolució.*

(i)

$$\frac{\frac{x+1}{x-2}-\frac{x-1}{x+3}}{\frac{x+2}{x+3}-\frac{x-1}{x+2}} = (\frac{x+1}{x-2}-\frac{x-1}{x+3}) : (\frac{x+2}{x+3}-\frac{x-1}{x+2})$$

$$= \frac{(x+1)(x+3)-(x-1)(x-2)}{(x-2)(x+3)} : \frac{(x+2)(x+2)-(x-1)(x+3)}{(x+2)(x+3)}$$

$$= \frac{7x+1}{(x-2)(x+3)} : \frac{2x^2+6x+1}{(x+2)(x+3)}$$

$$= \frac{(7x+1)(x+2)(x+3)}{(x-2)(x+3)(2x^2+6x+1)} = \frac{(7x+1)(x+2)}{(x-2)(2x^2+6x+1)}$$

$$= \frac{7x^2+15x+2}{2x^3+2x^2-11x-2}$$

(ii)

$$\frac{1-\frac{1}{x}}{1+\frac{1}{x}} = (1-\frac{1}{x}) : (1+\frac{1}{x}) = (\frac{x-1}{x}) : (\frac{x+1}{x}) = \frac{x(x-1)}{x(x+1)} = \frac{x-1}{x+1}$$

(iii)

$$\frac{\frac{x+3}{x+1}-\frac{x+4}{x-1}}{\frac{x+1}{x-1}+\frac{x+3}{x+1}} = (\frac{x+3}{x+1}-\frac{x+4}{x-1}) : (\frac{x+1}{x-1}+\frac{x+3}{x+1})$$

$$= \frac{(x-1)(x+3)-(x+4)(x+1)}{(x-1)(x+1)} : \frac{(x+1)(x+1)+(x-1)(x+3)}{(x+1)(x-1)}$$

$$= \frac{7x-7}{(x-1)(x+1)} : \frac{2x^2+4x-2}{(x+1)(x-1)}$$

$$= \frac{(7x+7)(x+1)(x-1)}{(x-1)(x+1)(2x^2+4x-2)} = \frac{(7x+7)}{(2x^2+4x-2)}$$