# Unit 1. Motion

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*Rest and movement are relative concepts: they depend on the chosen system of reference.*

## 1.- Definitions

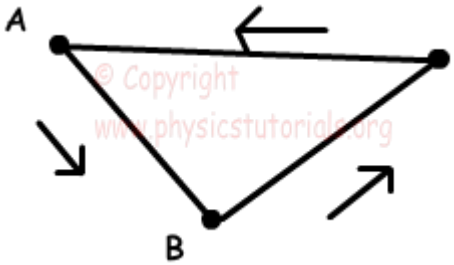
**System of reference:** System of reference is any definite point from where we observe the position of a moving object over a period of time.

**Position** is the place an object occupies with respect to the origin

**Trajectory** is the line made from joining all the points corresponding to the different positions of a moving object over a period of time

**Distance: Distance** is a scalar quantity representing the interval between two points. It is just the magnitude of the interval.

**Displacement: Displacement** can be defined as distance between the initial and final point of an object. It is a vector quantity having both magnitude and direction.




**Displacement is zero**

**Distance taken is=lABl+lBCl+lCAl**

**Speed: Speed** can be defined as “how fast something moves” or it can be explained more scientifically as “the distance covered in a unit of time”. Speed is a scalar quantity.

**Velocity:** It can be defined as “speed having direction” or displacement in a unit of time. Velocity is a vector quantity and it has both magnitude and direction.

**Acceleration:** We can easily define *acceleration* as “change in velocity”. This change can be in the magnitude (speed) of the velocity or in the direction of the velocity.

## 2.- Motion can be classified in this way

| Types of movement | trajectory | speed |
|---|---|---|
| Uniform linear motion | straight line | constant |
| Uniformly accelerated rectilinear motion | straight line | variable |
| Uniform circular motion | circular | constant in value<br>variable in direction |

## *2.1.- Uniform linear motion ULM*

- motion along a straight line
- with constant velocity or zero acceleration

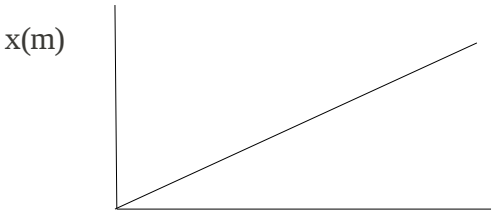
It can be described with only one equation:

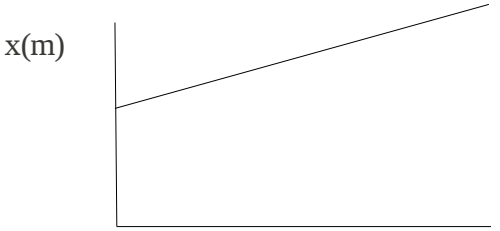
$$\mathbf{x = x_0 + v \cdot t}$$

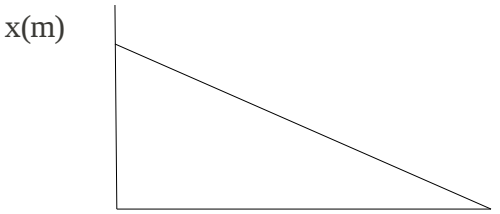
Where

x is the final position (m)
$x_0$ is the initial position of the object (m)
v is the speed (m/s)
t is the time (s)

Uniform linear motion graphs




**Exercises:**

1.- If the speed of a sound is 340 m/s, how far from us would a person be if it takes us 5 s to hear her cries?

*Sol: 1700 m*

2.- The Sun is 150 000 000 km from Earth. Calculate the minutes light takes to reach the Earth. The speed of the light is $3 \cdot 10^8$ m/s.

*Sol: 8.33 min.*

3.- An object is moving with a velocity 10 m/s in the negative direction of axis x. The object is initially at point A in x = 4 m . Find the position of the object at 5 s.

*sol: -46 m*

4.- An object is moving with a velocity of 23 m/s in the positive direction of axis x. The final position of the object is 8 m. Which was the initial position if the traveled time is 5 s?

*Sol: -107 m*

5.- An object is moving with constant velocity along axis x. The initial position is 13 m and the final position of the object is 5 m . Which is its velocity if the traveled time is 4 s?

*Sol: -2m/s*

6.-A man was driving his car from his office to his home at 50 km/h. Thirty minutes later he realized that he forgot some important documents at the office. What constant speed he should drive the car so that he can return to the office within 12 minutes?

*Sol: 125 km/h*

7.- Cyclist A starts off from an origin with a rectilinear trajectory and at speed of 72 km/h. At the same time, another cyclist B, placed 50 km in front of A, starts off in the same direction at a speed of 36 km/h. Where they find each other?

*Sol: x =100 km*

8. Train A leaves the station at the speed of 90 km/h. Five minutes later train B leaves the same station and travels in the same direction as train A at a speed of 120 km/h. Where and when they find each other?

*Sol:20 min, 30 km*

9.- Cyclist A starts off from an origin with a rectilinear trajectory and at speed of 72 km/h. At the same time, another cyclist B, placed 30 km in front of A, starts off in the opposite direction at a speed of 36 km/h. When and where they find each other?

*Sol: t =1000 s, 20 km*

10.-Train A leaves the station at the speed of 100 km/h. Ten minutes later train B leaves the same station and travels in the same direction as train A at a speed of 150 km/h. Where and when they find each other?

*Sol:30 min, 50 km*

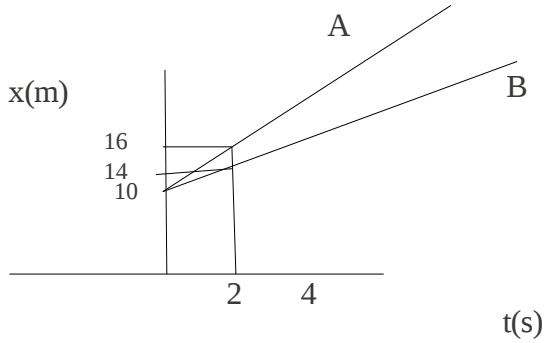
11. Plot a graph using the information below:

| x(m) | 0 | 1.5 | 4.5 | 6.0 | 9.0 |
|---|---|---|---|---|---|
| t(s) | 0 | 1 | 3 | 4 | 6 |

12.The equation of a object is x = 2 + 5 t. Answer:

a) type of movement
b) initial position
c) when the position of the object is 14 m
d) make the graph x-t

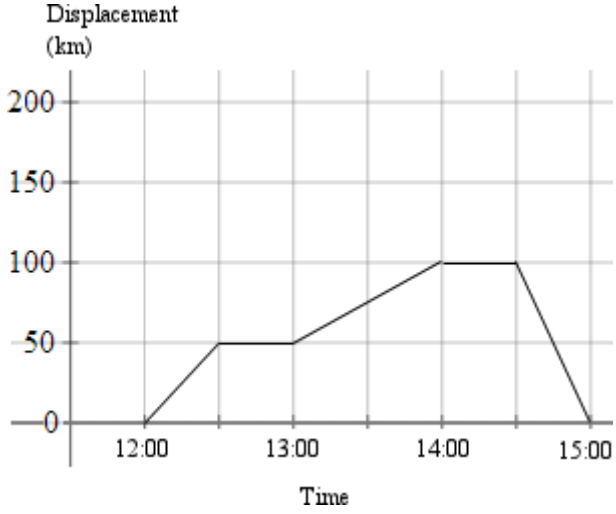
13.- The figure shows the graph x-t of two bodies, A and B, which start off from the same origin and at the same time
a) Which one moves faster?
b) Calculates the velocity of each one
c) Write the equation of each one
c) Where is the object A after 5 s? And the object B?




14.- A marathon runner runs at a constant 12 km/h.
a. Express the equation of the motion.

b. Graph the motion for $0 \leq t \leq 4$ h

15.- This is the graph of a journey by sports car:




a. What is the velocity for each stage of the journey?

b. What is the average (**mean**) velocity for the whole journey?

Sol: a) 100 km/h, 0, 50 km/h, 0, -200 km/h b) 66.7 km/h

16.- An object is moving with constant velocity 20 m/s in the positive direction of axis x for 50 s. Then the object is moving in the negative direction of axis x with constant speed 10 m/s for 70 s. Find the final position of the object and find the total travelled distance.

## *2.2.- Uniformly accelerated rectilinear motion UARM*

- motion along a straight line
- with variable velocity
- constant acceleration

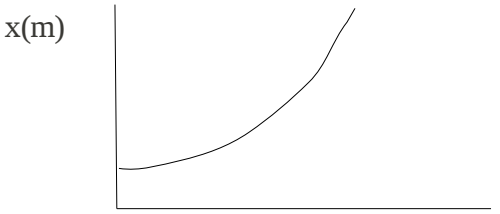
It can be described with three equations:

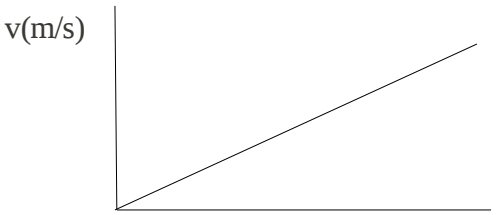
$$\mathbf{x = x_0 + v_0 \cdot t + \tfrac{1}{2}\, a \cdot t^2}$$
$$\mathbf{v = v_0 + at}$$
$$\mathbf{v^2 - v_0^2 = 2\, a\, (x - x_0)}$$

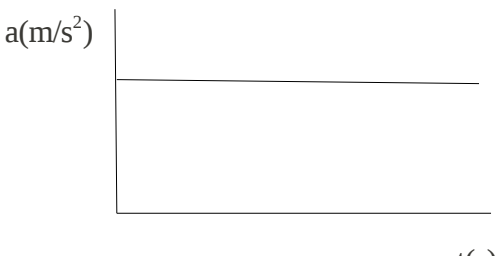
Where

x is the final position (m)
$x_0$ is the initial position of the object (m)
$v_0$ is the initial speed (m/s)
v is the speed in other moment (m/s)
t is the time (s)
a is the acceleration ($m/s^2$)

Uniformly accelerated rectilinear motion graphs



**Example:**
1. A car is initially travelling at 4 m/s then accelerates at 5 $m/s^2$ for 7 s. How fast is it going after this time?

Given:
$v_0 = 4$ m/s
$a = 5$ $m/s^2$
$t = 7$ s
$v = ?$

$v = v_0 + a \cdot t$
$v = 4 + (5) \cdot (7)$
$v = 4 + 35$
$v = 39$ m/s

**Exercises:**
1.- An object leaves a state of rest and moves with uniformly accelerated rectilinear motion. After 1 second it reaches a speed of 5 m/s.
a) What is the acceleration of the object?
b) How fast is it moving after 10 s?
c) How far has it travelled after 10 s?
d) How far has it travelled between seconds 9 and 10?

*Sol: a) 5 $m/s^2$ b) 50m/s c) 250 m d)47.5 m*

2.- An object moves with uniformly accelerated rectilinear motion and travels 100 m in 5 s. What is the acceleration?

*Sol: 8 $m/s^2$*

3.- A vehicle travels at a speed of 80 km/h. It accelerates and has a constant acceleration 2 $m/s^2$. How far will it have travelled after 30 s?

*Sol: 1566 m*

4.- An object leaves a position of rest and moves in a straight line with a constant acceleration of 0.2 $m/s^2$. How far will it travel in 1 minute?

*Sol: 360 m*

5. An object moving at 20m/s breaks at a rate of 3$m/s^2$. How long does it take it to stop completely?. What distance has the object travelled during this time?

*Sol.: t= 6,7s; x=66,6m*

6. A train leaves a station with an acceleration of 6$m/s^2$. What speed does it reach in 10s?. What distance has the train travelled during this time?

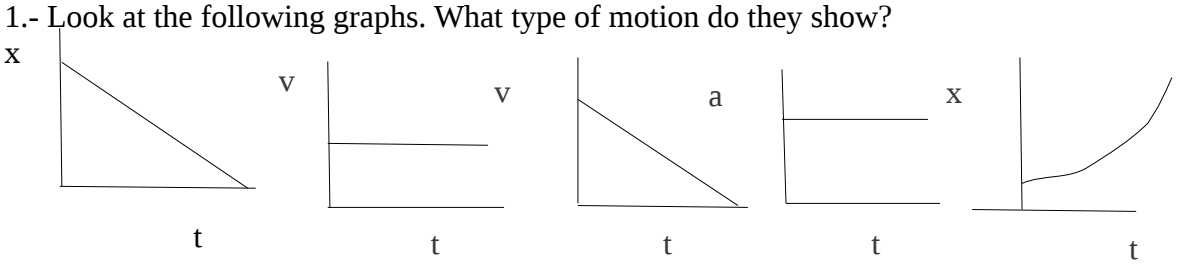
*Sol.:v=60m/s; x=300m*

7.-A train moving at a speed of 72km/h covers 150 m, from the moment it breaks until it stop. Considering a constant acceleration, calculate its value and the time it takes it to stop.

*Sol: -1,33$m/s^2$ t=15s*

**Exercises with graphs:**

1.- Look at the following graphs. What type of motion do they show?

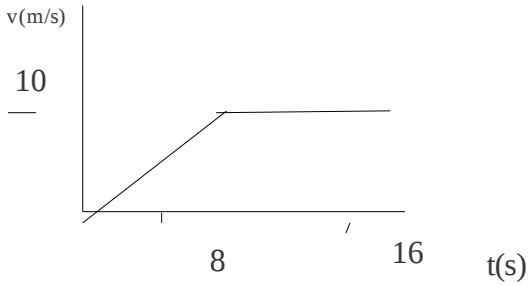



2.-Look at the graph and answer the following questions

a) What type of motion does the object have in each segment?

b) What is the acceleration of the object?

c) Calculate the distance travelled by the object

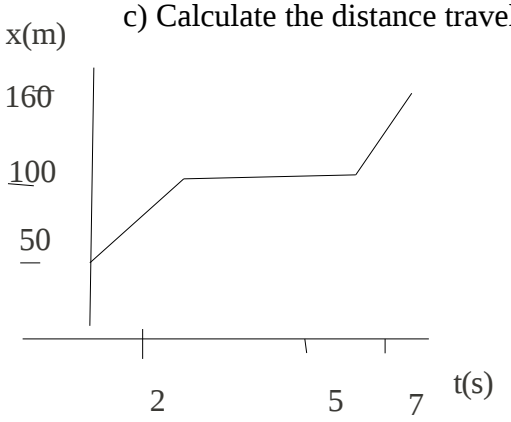



3.- Look at the graph and answer the following questions

a) What type of motion does the object have in each segment?

b) What is the speed of the object in each segment?

c) Calculate the distance travelled by the object




4.- Complete the table and make the graphs x-t and v-t

| t(s) | 0 | 1 | | 4 | |
|---|---|---|---|---|---|
| v(m/s) | 2 | 2,4 | 2,8 | | 6,4 |
| x (m) | 0 | | | | |

### *2.2.1.- Free Fall*

It is a type of uniformly accelerated rectilinear motion UARM

- motion along a straight line
- with variable velocity
- constant acceleration because of the gravity and it is always 9.8 $m/s^2$

It can be described with three equations:

$$\mathbf{h = h_0 + v_0 \cdot t - \frac{1}{2} g \cdot t^2}$$
$$\mathbf{v = v_0 - g \cdot t}$$
$$\mathbf{v^2 - v_0^2 = 2\,g\,(h_0 - h)}$$

Where

h is the final position (m)
$h_0$ is the initial position of the object (m)
$v_0$ is the initial speed (m/s). It can be positive if the object is vertically thrown up in the air, zero if the object is dropped and negative if the object is vertically thrown down.
v is the speed in other moment (m/s)
t is the time (s)
g is the acceleration of gravity, which value is 9.8 $m/s^2$.

**Example 1**: A penny is dropped from a roof top 72m above the ground. How fast is it going when it hits the ground?

Given:

| | |
|---|---|
| $h_0 = 72m$<br>$v = ?$<br>$g = 9.81\ m/s$<br>$v_0 = 0$<br>$h = 0$ | $v^2 - v_0^2 = 2g(h_0 - h)$<br>$v^2 - 0 = 2 \cdot 9.8 \cdot 72$<br>$v^2 = \sqrt{1412.64}$<br>$v = 37.6\ m/s$ |

**Exercises:**

1.-A firecracker is shot into the air. It needs to reach a height of 90m at the top of its flight. What does its initial velocity have to be to do this?

*Sol: 42.02 m/s*

2.-A bowler drops her bowling ball 1.2m onto her foot. How fast was the ball going when it hit?

*Sol: 4.85 m/s*

3. A stone is vertically thrown up with an initial speed of 40m/s. Answer the following: a) what is the maximum height it will reach? b)how long will it take for it to hit the ground again?.

*Sol: h = 81.6 m t=8,1s*

4.- How long will a body take to hit the ground if is dropped from a height of 54m?

*Sol.: t=3s*

5.- How long does a body take to hit the ground if it is thrown down with an initial speed of 10m/s from a height of 45m?

*Sol.: t=2,2s*

6.-. How long does a body take to hit the ground if it is thrown up with an initial speed of 10m/s from a height of 45m?

*Sol.: t=4,2s*

7.- A stone is thrown up from a bridge. Its initial speed is 6m/s. Calculate its speed when it hits the bridge again.

*Sol.: -6m/s*

## *2.3.- Uniform circular motion*

- motion of an object in a circle
- at a constant speed
- As an object moves in a circle, it is constantly changing its direction. An object undergoing uniform circular motion is moving with a constant speed. Nonetheless, it is accelerating due to its change in direction. So:

  $a_t = 0$ $a_n$ is constant and different from 0

It can be described with four equations:

$$\varphi = \varphi_0 + wt$$
$$w = 2\pi/T$$
$$v = w \cdot r$$
$$a_n = v^2/r$$

$\varphi$ is the angular final position (rad)
$\varphi_0$ is the angular initial position of the object (rad)
angular velocity (rad/s)
v is the speed (m/s)
t is the time (s)
T represents period (s)
r represents radius (m)
a represents the centripetal acceleration ($m/s^2$)

**Example:**
A motorcycle wheel spins at 1000 rpm. If its radius measures 50 cm, calculate: a)the angular speed in SI units; b)the linear speed; c) the period and the frequency; d) its number of complete turns in 15 s. *Sol.: 104,7 rad/s; 52,3m/s; T=0,03s f=33.3 Hz; 250 turns*

**Exercises:**

1. A body turns five times in two hours. Having a radius of 30cm, calculate: a) the angular speed in SI units; b) the linear speed of the wheel; c) the normal acceleration; c) the number of turns it makes in 50 minutes.

*Sol.: 0,0043 rad/s; v= 0,0013 m/s; a=5,7.$10^{-6}$ m/s; 2.05 turns*

2. A satellite which is 400 km above our planet traces a circular orbit round the Earth. The radius of the Earth is 6 378 km. If the satellite moves at a speed of 7 676 m/s, calculate how long it will take it to go around the Earth once.

*Sol.: 5 542 s*

3. A man walks around a roundabout with a perimeter of 628 m, and it takes him 6 minutes to go around once. Supposing he moves with uniform circular motion, calculate both his linear and his angular velocity.

*Sol.: v = 1.7 m/s, w =0.017 rad/s.*

4. A gladiator runs around the arena in the amphitheatre at a constant speed of 5 m/s. If it takes him 37.7 seconds to go round once, calculate the gladiator's angular speed, and the diameter of the arena.

*Sol.: 0.17 rad/s and 60 m*

5.- A record spins at 2,000 revolutions per minute. If its radius is 8 cm, calculate:

a) The distance a given point on the edge of the record has travelled in 5 seconds.
b) The time it took to go round an angle of $2\pi$ radians

*Sol.: s: 83.78 m t: 0.03 s*

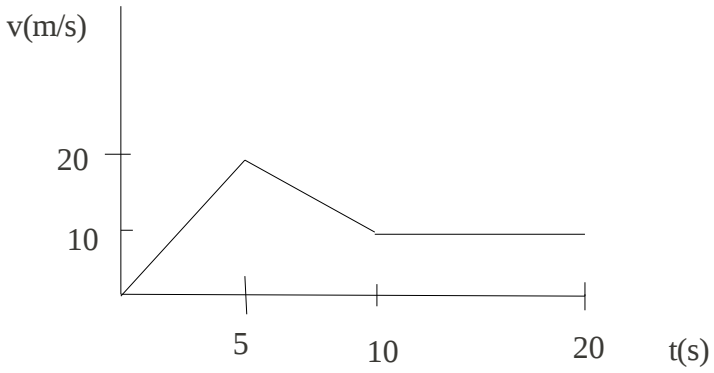
6. A wheel with a radius of 15 cm goes round three times in 0.5 seconds. Calculate:

a) Its angular velocity
b) Its centripetal acceleration
c) Its period and frequency.
d) The number of times the wheel would go round in 2 minutes.

*Sol.: a)37,6 rad/s b) 213.2 m/s2 c)0.167 s 6 Hz d)720*

# Practice exam

1. What type of movement, acceleration and distance has the object travelled in each segment? And the total distance travelled?




*Sol.:a = 4 m/s$^2$, -2 m/s$^2$, 0m/s$^2$ s= 50 m,75 m, 100m, 225 m*

2. An object, leaving a position of rest, reaches a speed of 20 m/s after travelling 50 m.

a) Which is the constant acceleration?
b) How long will it take to reach that speed?
c)How long does it take to reach a speed of 40 m/s?

*Sol.: a)4 m /s$^2$ b)5 s c) 10 s*

3. Plot a graph with the space-time and velocity-time of an object leaving a position of rest and accelerating with a =8m/s$^2$

4. If an object falls from a height of 20 m, what speed will it reach when it touches the floor? How long will it take to reach the floor?

*Sol.: a) 19.6 m/s b)2 s*

5. Calculate the angular velocity of the passengers in a big wheel which takes two minutes to complete a total round. Say the period, frequency and normal acceleration. R=10m.

*Sol.: a)0.052 rad/s b) 120 s c)0.0083 Hz s d) 0.027 m /s$^2$*

6. A bus takes a uniform linear motion at a speed of 30 km/h and it is 150 m far from the bus stop. A boy who is in a position of rest talking to his friends sees the bus and runs to the bus stop which is 250 m away with an acceleration of 2 m/s$^2$. Will he catch the bus?

*Sol.: a) yes*