Chapter 3
KINEMATICS

GOALS

When you have mastered the content of this chapter, you will be able to achieve the following goals:

Definitions

Use the following terms to describe the physical state of a system:

displacement
velocity
uniform circular motion
acceleration
uniformly accelerated motion
radial acceleration
projectile motion
tangential acceleration

Equations of Motion

Write the equations of motion for objects with constant velocity and for objects with constant acceleration.

Motion Problems

Solve problems involving freely falling and other uniformly accelerated bodies, projectile motion, and uniform circular motion.

Acceleration Effects

List the effects of acceleration on the human body.

PREREQUISITES

Before beginning this chapter you should have achieved the goals of Chapter 1, Human Senses, and Chapter 2, Unifying Approaches. You must also be able to use the properties of right triangles to solve problems.

Mathematics Self-Check

If you can solve the following problem easily and correctly, you are prepared for this chapter:

A surveyor wishes to determine the distance between two points ***A*** and *B*, but he cannot make a direct measurement because a river intervenes. He steps off a line *AC* at a 90^o; angle to *AB* and 264 meters long. With his transit, at point *C* he measures the angle between line *AB* and the line formed by *C* and *B*. Angle *BCA* is measured to be 62^o. What is the distance from *A* to B? [497 m]

If you had difficulty getting this answer, you will find additional information in Section A.6, Right Triangles, of the Appendix.

Chapter 3
KINEMATICS

3.1 Introduction

For the greater part of your life, you have been engaged in the process of getting from here to there. First you learned to crawl, then to walk, and later to run. These are examples of motion and change of position. In these motions you were concerned with distances, directions, rates of motion, and time, or duration, of motion. This same concern with motion is true for change of position by a mechanical device such as a bicycle, an automobile, or an airplane. How would you describe your present state of motion?

How would you describe the motion of an Olympic sprinter? What would be your description of a professional figure skater's motion when she does a spin on ice skates? Have you seen pictures of an astronaut moving about in the "zero gravity" environment of space? What concepts do you need to describe the astronaut's motion? You will be introduced to the concepts of motion in various forms in this chapter. This study of motion (without concern for its causes) is known as *kinematics*.

3.2 Characteristics of Distance and Displacement

In order to develop the relationships and characteristics of motion, it is necessary for us to define some terms. If a body is moved from one place to another, it is said to be displaced. This *displacement* is specified by both *magnitude* and *direction*. If you move your coffee cup along the table top 10.0 cm to the east and then 10.0 cm to the north, you will have displaced your cup 14.1 cm to the northeast. The coffee cup will have traveled a distance of 20.0 cm and will be a distance of 14.1 cm from its starting point. You will notice that *distance* has only magnitude. Such a physical quantity is called a *scalar* quantity. A scalar quantity is completely specified by a number and its proper dimensional unit. Can you think of other scalar quantities with which you are familiar? A quantity such as displacement, that is only completely determined when you have given *both* its magnitude and its direction is called a *vector*. A *vector* quantity can be represented graphically by an arrow in which the shaft of the arrow represents the line of action and the arrow head is the direction of action along the line. Vector A will be shown in *boldface* type **A**. The *length* of the line gives the *magnitude* of the vector and is represented in the usual type style A. Thus a displacement of 8 km northeast is represented by a vector making an angle of 45° with the easterly direction and 8 units long (see Figure 3.1).

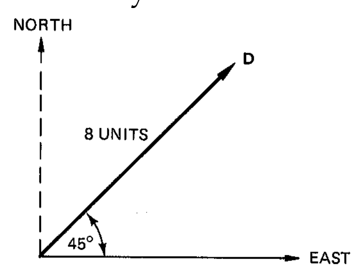


FIGURE 3.1
Vector representation of a displacement of 8 km northeast of origin.

Suppose you ride a bicycle from your home to the physics building, a straight line distance of 2 km. The total distance you traveled was 2 km. After you ride back home, you will have traveled a distance of 4 km, but your net displacement is zero (Figure 3.2). The addition of distances (scalars) follows the usual rule of addition. The addition of displacements (vectors) must take into account the directions of the displacements involved. In this example the first displacement (from your home to the physics building) and the second displacement (from the physics building to your home) are in opposite directions. The addition of these two vector displacements gives a zero net displacement.

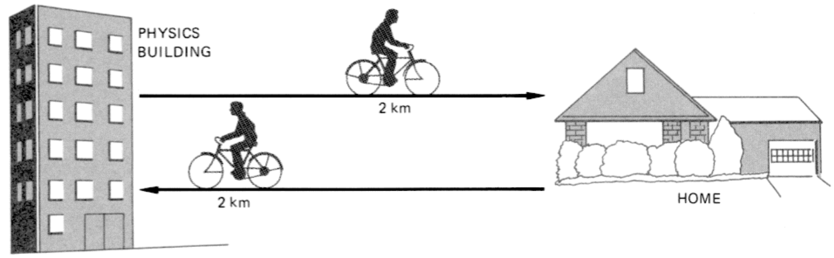


FIGURE 3.2
Graphical representation indicating how one can travel 4 km and have zero displacement.

Vectors do not obey the simple algebraic properties of scalars. For example, when you add the two scalars, 2 plus 2, you obtain 4. If you add two vectors, both of magnitude three, you may obtain any number from 0 to 6 for the magnitude of the sum of these two vectors (see Figure 3.3). Add the two displacements 3 km east and 3 km east. What is the net displacement result? If you said 6 km east, you got it. Now add the two displacements, 3 km east and 3 km west. What is the net displacement? If you said 0 km, you are right. How can you add two displacements, each of which has a magnitude of 3 km, and obtain a final displacement whose magnitude is 3 km?

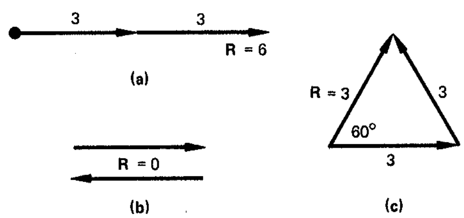


FIGURE 3.3
Three simple vector additions: (a) parallel in same direction, (b) anti-parallel, and (c) at 60° for the case of equal magnitudes.

3.3 Graphical Method for Adding Vectors

Suppose you dropped a contact lens from your eye and it rolled across the tile floor. It rolled 10.0 m across the floor in a direction 37° north of east. There it struck the wall and bounced 8.0 m in a direction 30 ° west of north. What is the displacement of the contact lens? To use the graphical method for adding vectors, we represent the first displacement by an arrow pointing 37 ° north of east and scaled to represent 10.0 m in length, as we have drawn vector **A** in Figure 3.4a. We represent the rebound displacement by the vector **B**. To add the vectors **A** and **B**, we draw the vector **B** extending from the tip of vector **A** as shown in Figure 3.4c.

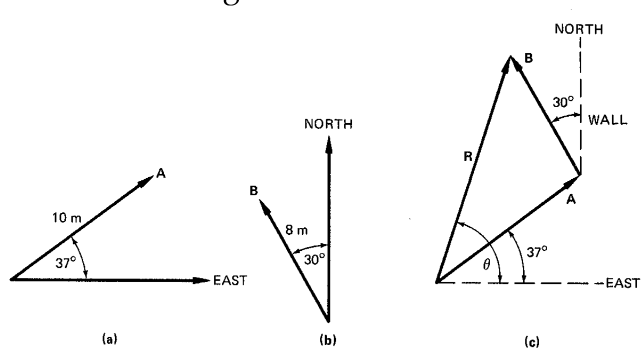


FIGURE 3.4
Vector representation of two displacements, **A** and **B**. (c) The resultant **R** of displacements **A** and **B**.

The sum of vector **A** and vector **B** is called the *resultant* and is shown by the vector **R**. The magnitude of the resultant displacement **R** can be measured with a ruler, and the angle between **R** and east can be measured with a protractor. For this example **R**, the final location of the dropped lens, is found to be given by a vector 13.7 m at an angle 73 ° north of east. Now suppose we wish to add vector **C** to vectors **A** and **B** given above. Vector **C** has a magnitude of 6.0 m and points to the west. We draw vectors **A** and **B** as above and then from the tip of **B** draw vector **C** as shown in Figure 3.5. We can find the resultant **R**, or the vector sum, of **A** + **B** and **C** by drawing a vector from initial point of **A** to the tip of **C**. We can obtain the magnitude of **R** by scaling, that is, by measuring the length and the direction of **R** by measuring the angle from the east-west reference axis with a protractor. Then we have all the data needed to define the vector **R**, direction and magnitude. For this example the vector **R** is given by a displacement of 13.1 m in a direction 81° north of west. This procedure can be used for addition of any number of vectors.

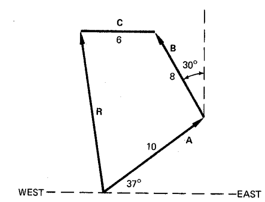


FIGURE 3.5
Graphical representation of the addition of three vectors (displacements).

You can find the difference between two vectors by using the same procedure. To find the value of the vector (**A** - **B**), you add the vector **-B** to the vector **A**. The vector **-B** has the same magnitude as the vector **B** but the opposite direction. The vector (**A** - **B** is shown in Figure 3.6. The method for the finding of the magnitude of the vector **R** and its direction is as given above. For this case the vector **R** is given by a vector about 7&xf8; south of east with a magnitude of 12.0 m. In Section 3.2 we asked how you might add two displacements, each of 3 km magnitude and obtain a final displacement of 3 km. From this graphical method of adding vectors, you see that the resultant R and the two vectors **A** and **B** form a triangle. If the three vectors **A, B, R** each have a length 3, then the vectors, **A, B** and **R** must form an equilateral triangle. Hence vector **R** makes an angle of 60 ° with vector **A** (see Figure 3.3c).

3.4 An Algebraic Method for Adding Vectors

Another way of designating the displacement of the contact lens dropped in the previous section is to specify the number of the floor tile on which it is lying from a designated corner of the room. This method of locating a point in a plane with two *coordinates* is the visual technique incorporated in the cartesian model of space.

By analogy, the cartesian model enables us to state the position vector of the final displacement of the contact lens (from the designated corner) as the sum of a north component vector and an east component vector. In our example the lengths of these component vectors would have units of tile length. In general, the vector in a plane can be specified by its horizontal x and vertical y components in any chosen coordinate system. For instance, vector **A** given in Figure 3.5 has an east- west component of **A**cos 37 ° and a north-south component of **A**sin 37°. Let us make the substitution of the x-axis for the east-west direction and the y-axis for the north- south direction and proceed to find the resultant of **A**, **B**, and **C** in Figure 3.5. First we find the x component and the y component of each vector **A**, **B**, and **C**. The x component of the resultant **R** is equal to the algebraic sum of the x components of **A**, **B**, **C,** and the y component of **R** is equal to the algebraic sum of the y components of **A**, **B**, and **C**. The method is outlined in Table 3.1:

TABLE 3.1

Vector	x Component	y Component
A	$A \cos 37° = 10.0\ (0.800) = 8.00$	$A \sin 37° = (10.0)(0.600) = 6.00$
B	$B \cos 120° = (8.00)(-0.50) = -4.00$	$B \sin 120° = (8.00)(0.866) = 6.93$
C	$C \cos 180° = (6.00)(-1.00) = -6.00$	$C \sin 180° = (6.00)(0.00) = 0.00$
R	$R \cos \theta = (8.00 - 4.00 - 6.00)$ $= -2.00$	$R \sin \theta = (6.00 + 6.93 + 0.00)$ $= 12.93$

The magnitude of **R** is found by using the pythagorean theorem,

$$R = \sqrt{[(2.00)^2 + (12.9)^2]}$$

$$R = 13.1 \text{ m}$$

We can find the direction of **R** by using the definition of the tangent of an angle,

$$\tan\Theta = R\sin\Theta / R\cos\Theta = 12.9 / -2.00 = -6.46$$

$$\Theta = 98.2^{o}$$

In this case, the resultant vector **R**, which is the sum of **A**, **B**, and **C** is given by a vector of length 13.1 m in a direction 98.8^{o} counterclockwise from the *x*-axis.

3.5 Characteristics of Motion

In the above examples please notice that the displacement and the distance traveled may be given by different numerical values. In understanding problems of motion it is very important to have clearly in mind what information you have been given: Is it a distance or a displacement? What you are seeking: Is the answer to be a scalar or a vector?

In many cases we are interested not only in whether a body has moved but also in how fast the body moved. If we measure how much time is required to move an object a given distance or through a given displacement, we can calculate the rate of change of distance with time or the time rate of change of displacement. *Speed* is defined as the time rate of change of the distance traveled. Since both distance traveled and time are scalars, speed is a scalar quantity with the dimensions of length divided by time, or, for our purposes, with units of meters divided by seconds. Velocity is defined as the time rate of the change in displacement. The average velocity shown by the symbol v_{ave} is found as follows:

$$v_{ave} = (s_1 - s_0) / (t_1 - t_0) = \Delta s / \Delta t \qquad (3.1)$$

where s_0 is the displacement of the body at time t_0 and s_1 is the displacement of the body at a later time . We have used Δs to represent the change in displacement that occurred in the time of Δt.

Since velocity is the ratio of the change in displacement (a vector) to the change in time (a scalar), velocity is a vector quantity. What are the dimensions of velocity, and what units does it have? You may notice that velocity is the ratio of a quantity measured in meters (displacement) to a quantity measured in seconds. *Instantaneous velocity* is velocity at any given instant in time and is discussed in the Section 3.11. Can you change the velocity of a moving object without changing its speed? If you change only the direction of the velocity of an object and not its magnitude, then the speed, the magnitude of velocity, does not change. If you are walking along the sidewalk with a velocity of 3 km/hr east, and turn a corner to go 3 km/hr north, what is your speed? Your speed remains the same 3 km/hr, but your velocity has changed from 3 km/hr east to 3 km/hr north.

The simplest motion that we can have is that of constant velocity. That means neither the direction nor the magnitude of the time rate of change of the displacement is changing. This is motion in a straight line at a constant rate. One example is walking at a rate of 5 km/hr east. The displacement that occurs when a body is moving with constant velocity is computed from the equation

$$s = v_{ave}\, t \tag{3.2}$$

Another simple kind of motion is to travel at a constant speed. The direction of displacement may change, but the time rate at which the distance traveled changes is constant. An example of such motion is traveling along the highway at 89 km/hr (55 mph). In this kind of motion the distance traveled is given by the product of the speed and the time of travel.

You recognize the difficulty in always traveling with either constant velocity or constant speed: It does not permit you to stop moving if you are moving already or to start moving if you are presently at rest. Clearly, then we need to consider other kinds of motion in which the velocity changes.

3.6 Linear Motion

To begin let us simplify our discussion of motion with changing velocity by restricting it to motion of objects along a line. This includes a number of common experiences such as a runner on a track, an automobile on the highway, or a toy car rolling down an inclined table. In these cases, the object is moving either forward or backward, either away from or toward the starting point. Hence, velocity can have only two directions which we can designate as positive and negative. In these common situations, you notice that the difference between velocity and speed appears to be of minor importance; only the sign may be different. So you can understand why in ordinary conversation the distinction between velocity and speed is not carefully preserved. However, the sign in front of the magnitude of the velocity is highly significant. It tells you whether an object is going forward or backward, up or down, right or left, depending upon the direction that you have chosen as positive.

When a body starts moving from rest, its velocity changes. If we choose forward as the positive direction, as you back your car out of the garage you decrease the velocity of your auto, that is, you start from rest ($\mathbf{v} = 0$), and give it a negative (backward) speed. As you start your car forward down the street, you increase the velocity of your car. The change of the velocity of an object in a unit of time is called the *acceleration*. The average acceleration is given by the equation

$$a_{ave} = (v_1 - v_0)/(t_1 - t_0) = \Delta v/\Delta t \tag{3.3}$$

where v_0 is the velocity at t_0 and v_1 is the velocity at a later time t_1, and where Δv represents the change in velocity during the time interval Δt. Since velocity is a vector, the acceleration is also a vector quantity. What are the dimensions of acceleration, and what units does it have? You may notice that acceleration is given by the ratio of a quantity measured in meters divided by seconds to a quantity measured in seconds. Instantaneous acceleration is the acceleration at any given instant in time and is treated in the Section 3.11 of this chapter.

Questions

Figure 3.7 shows a plot of velocity as a function of time for constant acceleration. Study the curve and answer the following questions:

1. What does the intercept *C* on the velocity axis represent?
2. What does the slope of *CE* represent?
3. What does *DE* represent?
4. What does *tE* represent?
5. Note that the area of *OtDEC* is made up of a rectangle *OtDC* and triangle *CDE*. What is the area of *OtDC*, and what does it represent?
6. What is the area of *CDE*, and what does it represent?
7. What is the total area *OtDEC*, and what does it represent?

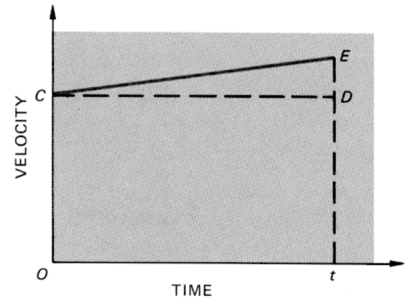


FIGURE 3.7
A velocity-time graph for constant acceleration.

Answers

1. original velocity, v_0
2. acceleration, *a*
3. change in velocity, Δv
4. velocity at time *t*, v_t
5. displacement for constant velocity v_0 and time *t*
6. displacement in time *t* resulting from the change in velocity
7. total displacement in time *t*

3.7 Uniformly Accelerated Motion

Let us develop the relationships for motion in which the time rate of change of the velocity, the acceleration, is constant. This is, of course, an idealization since in no real system is it possible to keep the rate of change of the velocity a constant for all times. But the motion of many systems approximates this idealization. For example, you pull away from the curb in your automobile, and after one second you are traveling at a forward speed of 10 km/hr (6 mph). Then after two seconds you are traveling at +20 km/hr (12 mph), after three seconds at +30 km/hr (19 mph), and so on. The rate at which you are changing your velocity is a constant. That is, the acceleration is constant and has the value of (30 km/hr - 0 km/hr)/(3 sec), or +10 km/hr per second. This is known as uniformly accelerated motion. From our definition of acceleration we get

$$a = \Delta v/\Delta t = (v_f - v_0)/\mathrm{t} \tag{3.3}$$

or solving for v_f gives

$$v_f = v_0 + at \tag{3.4}$$

in which v_f is the velocity at any time *t*, if the original velocity is v_0 and the acceleration *a* is constant. If the acceleration is constant, the average velocity v_{ave} is given by one-half of the sum of the final and initial values of velocity,

$$v_{ave} = (v_f + v_0)/2 \tag{3.5}$$

For the specific example above the average velocity is to be (30 km/hr + 0 km/hr)/2 = +15 km/hr. The change in displacement during time *t* is given by the product of the average velocity times the time,

$$\Delta s = s - s_0 = v_{ave}\, t = (v_f + v_0)/2)t \quad (3.6)$$

For the automobile pulling away from the curb above, the displacement, or forward distance traveled, is 15 km/hr • 3 sec. Converting kilometers to meters and hours to seconds, we obtain (15,000 m/hr)(1 hr/3600 sec) (3 sec) = +12.5 meters. Substituting the value of v_f from Equation 3.4 in Equation 3.6, we obtain an expression for the displacement in terms of the initial displacement, initial velocity, acceleration, and time:

$$s - s_0 = v_0 t + at^2/2 \quad (3.7)$$

Using the example of the automobile, we choose the original displacement at the curb as the position where $s_0 = 0$. The starting velocity v_0 is zero since the auto starts from rest. Since we found the acceleration to be given by 10 km/hr/sec, after 3 sec the displacement is given by

$$s - 0 = 0(3) + 1/2\,(10 \text{ km/hr-sec})(3 \text{ sec})^2$$
$$= +45 \text{ km-sec/hr} = 45{,}000 \text{ m}/3600$$
$$= +12.5 \text{ m}$$

Another equation for *uniformly accelerated motion* is obtained by eliminating time from Equations 3.4 and 3.7 to obtain an expression for the velocity as a function of acceleration and distance.

$$2a\,(s - s_0) = v_f^2 - v_o^2 \quad (3.8)$$

This product of two vectors is known as a scalar product since it yields a scalar quantity. It will be discussed in Section 5.2. In this section, since all the vectors are along the same line, this product can be treated as the usual algebraic multiplication. If you consider the initial position as the origin, or zero displacement, then the three basic equations of *uniformly accelerated motion* become:$v_f = v_o + at$

$$s = v_o t + at^2/2$$
$$2a \bullet s = v_f^2 - v_o^2 \quad (3.9)$$

Starting from rest is a special case in which the initial velocity is zero, $v_o = 0$. If the initial displacement is also zero, the equations can be reduced to the following shortened forms: $v_f = at$

$$s = at^2/2$$
$$2a \bullet s = v_f^2 \quad (3.10)$$

when the initial velocity is zero and the initial displacement is zero.

Consider a low-friction toy car rolling down a slightly inclined table. Shown below in Table 3.2 and Table 3.3 are the experimental data. Compute the missing items in the table. Can you determine what type of motion is represented by this physical situation? Experimental data for a 53.6 g toy car rolling down an incline are given in Table 3.2. The experiment was repeated with a 50 g mass added to the toy car, and the data in Table 3.3 were obtained.

Hot wheels toy car of mass 53.6 gm

TABLE 3.2
Table of Data and Calculations

Time	Distance Down the Incline	Average Velocity	Average Acceleration
1 sec	12.4 cm	______	______
2 sec	49.2 cm	______	______
3 sec	111.8 cm	______	______
4 sec	198.8 cm	______	______
5 sec	310.0 cm	______	______

Hot wheels toy car with 50 gm mass on top to it (total mass = 103.6 gm)

TABLE 3.3
Table of Data and Calculations

Time	Distance Down the Incline	Average Velocity	Average Acceleration
1 sec	12.0 cm	______	______
2 sec	48.3 cm	______	______
3 sec	109.8 cm	______	______
4 sec	198.2 cm	______	______
5 sec	310.0 cm	______	______

What can you say about the influence of mass on the motion of the car down the incline? Since mass is a measure of the inertia, the tendency to resist changes in motion of an object, how do you explain the fact that although the mass of a moving object is almost doubled, the data are changed very little?

EXAMPLE

An automobile starts from rest and acquires a forward velocity of 36 km/hr in 5 sec. What is its acceleration, and what is the change in position during this time? Take the forward direction as positive.

$$v_o = 0$$

since the car starts from rest.

$$v_f = +36 \text{ km/hr} = 10 \text{ m/sec}$$

$$t = 5 \text{ sec}$$

How do you find the acceleration? (Hints: 1 km = 1000 m; 1 hr = 3600 sec.) From Equation 3.3

$$a = \Delta v / \Delta t = (v_f - v_0)/t = 10 - 0/5 = +2 \text{ m/sec}^2 \qquad (3.3)$$

From Equation 3.10

$$s = at^2/2 = (2)\,(5)^2/2 = +25 \text{ m} \qquad (3.10)$$

Does this example describe a realistic situation? Plot a curve similar to the one in Figure 3.7 for the sample problem worked above. Then plot the displacement as a function of time. What type of curve did you get? Which equation describes the curve?

There are many examples of almost uniformly accelerated motion, but perhaps the most familiar example is a body falling freely through the air when the air resistance is neglected. For such an idealized falling body the acceleration a is constant and is directed vertically downward, that is, follows the direction of a plumb bob line. This constant downward acceleration is called the *acceleration due to gravity* and is designated by g. All of the equations developed above for uniformly accelerated motion apply for an ideal falling body. One normally replaces a by g, which has a numerical value of about 9.80 m/sec^2 downward near the surface of the earth. You can get an approximate value for the magnitude of g by the following simple experiment. Toss a ball straight up, estimate the time the ball is in flight and the height to which the ball is thrown above your hand. You may estimate the time by counting "thousand-and-one, thousand-and-two ..." Each count is approximately one second. You should be able to estimate the height in meters that the ball rises. You then can calculate the value of g by dividing twice the height by the square of one-half the time of flight. From Equation 3.10, we know that

$$h = 1/2g(t/2)^2$$

Solving this for g,

$$g = 2h/(t/2)^2 = 8h/t^2 \text{ m/sec}^2 \qquad (3.11)$$

What value did you get? ____________ m/sec^2. In any case you will find your value to be nearer 10 m/sec^2 than to either 1 or 100 m/sec^2. Thus your determined value is the proper order of magnitude.

EXAMPLES

1. A person hangs from a diving board so that his feet are 5 m above the water level in the pool. He lets go of the board. Assuming idealized falling motion,how much-time passes before his feet strike the water, and what is their velocity at that time?

Given: If we take downward as the positive direction, then s = +5 meters down from the board. At the beginning (t = 0), the person is at rest. This implies v_0= 0. Then the person begins to fall with an acceleration of g = +9.80 m/sec^2 (positive downward).

Find:

a. time of fall = t

b. velocity = v_f

Relationships: $s = v_o t + at^2/2$

$v_f = v_o + at$

Substituting numerical values,

$$5m = 0t + (1/2)(9.80 \text{ m/sec}^2)\, t^2$$

$$t^2 = 10/9.80 \text{ sec}^2/\text{m} \quad t = 1.01 \text{ sec}$$

$$v_f = 0 \text{ (m/sec)} \times 1.01 \text{ sec} + 9.80 \text{ m/sec}^2 \bullet 1.01 \text{ sec}$$

$$v_f = 9.80 \text{ m/sec downward}$$

2. A ball is thrown vertically upward with a velocity of 30 m/sec. Assuming idealized motion, how high will it rise, and when will it reach its peak of flight? How long will it be before it returns to the starting point, and what will the velocity be at that time

Given: v_o = +30 m/sec (positive upward). At its peak the ball stops; this implies that the speed at the peak is zero, v_{peak} = 0 when t_{peak} = time to reach the peak. Then the ball starts downward with $g = -9.80\ \mathrm{m/sec^2}$.

Find:

a. time of rise = t_{peak} = time in flight/2
b. height = s
c. final speed = v_f

Relationships: $v_f = v_o + gt \qquad s = v_o t + gt^2/2 \qquad 2g \bullet s = v_f^2 - v_o^2$

Using the first of these and inserting values for the final speed v_f, the original speed v_o and g, we get

$$0 = 30.0\ \mathrm{m/s} - 9.80\ \mathrm{m/sec^2}\ t_{peak}$$

and solve for t_{peak}. Note that if the original velocity is positive upward, g is then negative (downward).

$$\text{time to reach the peak} = t_{peak} = 30.0\ \mathrm{m}/9.80\ \mathrm{m/sec^2} = 3.06\ \mathrm{sec}$$

Substitute this value for the time into the equation for the displacement:

$$s = v_o t + gt^2/2; \qquad s = 30\ \mathrm{m/sec} \bullet 3.06\ \mathrm{sec} - 1/2\ (9.8\ \mathrm{m/sec^2})(3.06)^2$$

$$\text{height} = 91.8\ \mathrm{m} - 45.9\ \mathrm{m} = +45.9\mathrm{m}$$

or using the other equation

$$2a \bullet s = v_f^2 - v_o^2 \qquad (3.9)$$

$$+2\ (-9.80\ \mathrm{m/sec^2})\ s = 0^2 - (30\ \mathrm{m/sec})^2$$

$$s = 900\ \mathrm{m^2/sec^2}/19.6\ \mathrm{m/sec^2} = +45.9\ \mathrm{m}$$

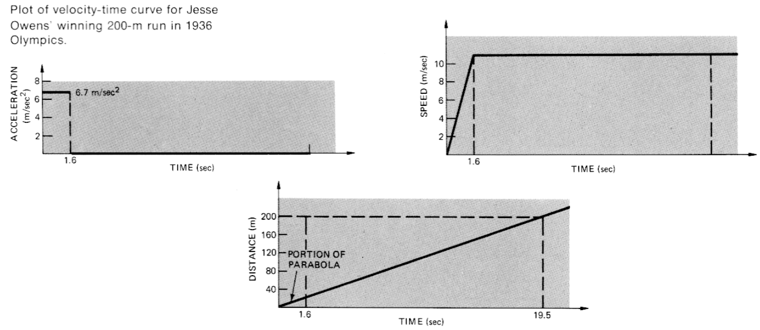
It will take the ball the same time to fall as to rise; so total time of flight = 2(3.06 sec) = 6.12 sec. Therefore, the final velocity

$$v_f = 30\ \mathrm{m/sec} - 9.80\ \mathrm{m/sec^2} \bullet 6.12\ \mathrm{sec}; \qquad v_f = -30.0\ \mathrm{m/sec}$$

Note the negative sign. When the ball returns, it will be going down (negative), with the same speed with which it started up.

3. A record run in the 200-m dash by Jesse Owens in the 1936 Olympics is well approximated by assuming that Owens started from rest and accelerated at the constant rate of $6.7\ \mathrm{m/sec^2}$ for a time of 1.6 sec. He then ran the remainder of the race at a constant speed. Draw a graph of Owens' acceleration as a function of time. Draw a graph of his speed as a function of time. Draw a graph of the distance he has run as a function of time.
[See Figure 3.8 for the solution.]

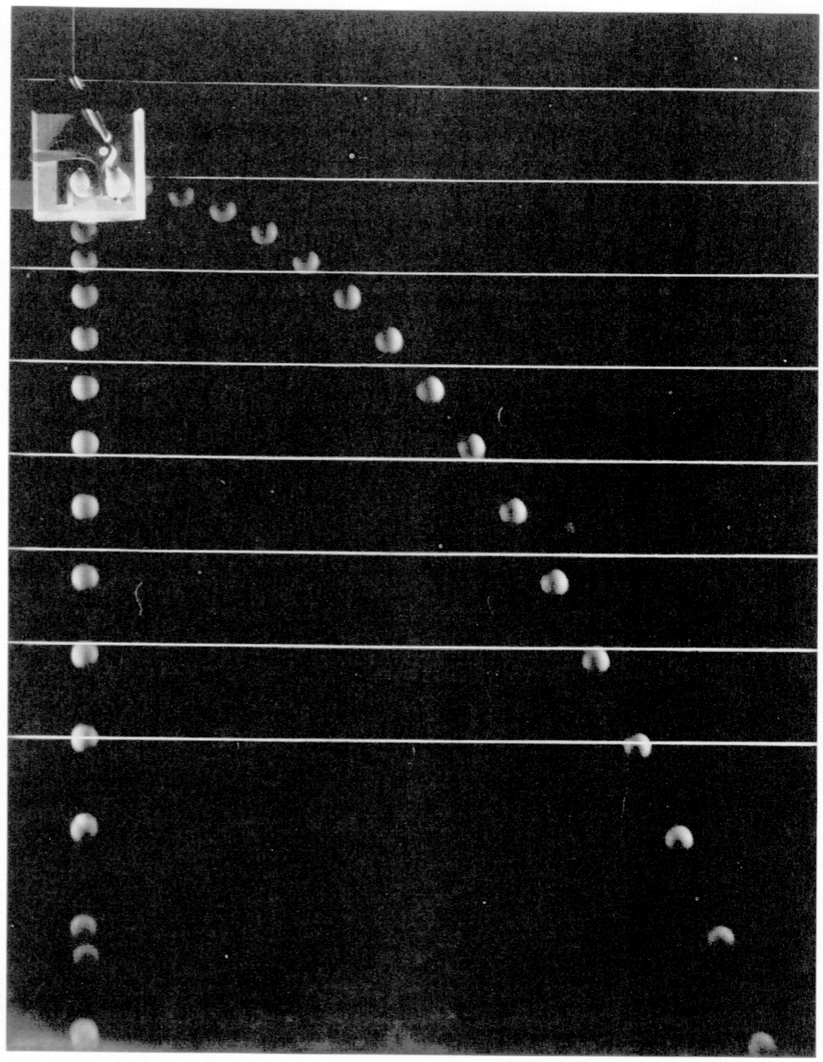
FIGURE 3.8
Plot of velocity-time curve for Jesse Owens' winning 200-m run in 1936 Olympics.



3.8 Projectile Motion

Your friend throws you a tennis ball, which you catch and return to her. The motion of the ball is motion in a vertical plane. It is a common type of motion, but it is more complicated than linear motion. It is motion in which the object has an almost constant velocity in one direction and has almost uniform acceleration in a direction at right angles to the constant velocity (see Figure 3.9). This type of motion is called *projectile motion*.

Two golf balls were released simultaneously, one dropped freely and the other one given a horizontal velocity of 2 m/sec. They were photographed at 1/30-sec intervals. The ball that was dropped executes uniformly accelerated motion, and the one with an initial horizontal velocity executes projectile motion. The horizontal white lines in picture are a series of strings placed behind the golf balls at 6-inch intervals. From this you should be able to scale the photograph and determine the vertical velocity for any interval and also the horizontal velocity. (Picture from *PSSC Physics*, D.C. Heath and Company, Lexington, Mass., 1965.)



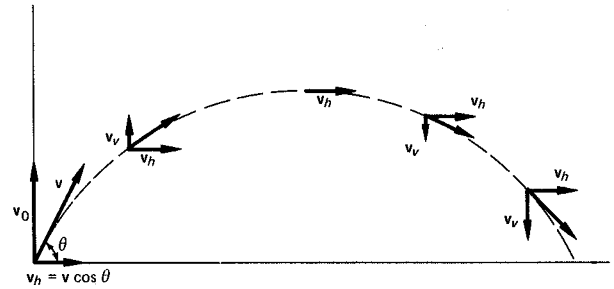
The tennis ball, when we neglect the effects of spin and air resistance, is moving with a constant velocity in the horizontal direction. (The horizontal components of motion are shown in the figure by the subscript h.) It has the acceleration due to gravity in the vertical direction.(The vertical components of motion are shown by the subscript v.) We shall treat projectile motion as two separate sets of scalar equations using only positive and negative signs to indicate directions, up and down in the vertical direction or forward and backward in the horizontal direction.

Because the motion in the horizontal direction in this idealized case is motion of constant velocity, the horizontal displacement of the tennis ball is given by the product of the horizontal velocity and the time.

$$s_h = v_h\, t \qquad (3.12)$$

where v_h is the horizontal velocity and t is the time. This expression neglects both air resistance and spin, and so v_h is a constant since the acceleration in the horizontal direction is zero.

FIGURE 3.9
The flight of a projectile fired at an angle of θ above the horizontal. The horizontal and vertical components of the velocity are shown at several positions of the flight.



In the vertical direction, the equations of uniformly accelerated motion Equation 3.9 hold true. That is, where v_v = vertical velocity at time t, v_o is the vertical velocity at $t = 0$, and s_v is the vertical displacement.

$$v_v = v_o + gt \qquad [a] \qquad (3.13)$$

$$s_v = v_v\, \mathrm{t} + 1/2\, gt^2 \qquad [b]$$

$$v_v{}^2 = v_o{}^2 + 2gs_v \qquad [c]$$

Note that if we choose the upward direction as positive, then acceleration due to gravity is g= -9.80 m/sec^2.

Suppose a projectile is fired with a velocity of v at an angle Θ to the ground. The horizontal component is constant during flight but the vertical component is changing because the acceleration is constant in a downward direction: g = -9.80 m/sec^2. The horizontal component of velocity is $v_h = v \cos \Theta$, and the original vertical component of velocity is $v \sin \Theta$. Hence the vertical component at any time t after firing is

$$v_v = v \sin \Theta + gt \qquad (3.14)$$

What is the vertical component of velocity at peak of flight? At the peak of its flight the projectile is moving only in a horizontal direction, so the vertical component of its velocity is zero. Substituting zero for v_v in Equation 3.13a and solving for the time required to reach the peak of flight, we get the following algebraic equations:

$$v_v = 0 = v_o + g\, t_{peak} = v \sin \Theta + gt_{peak} \qquad (3.15)$$

$$t_{peak} = -\, v \sin \Theta \,/ g \qquad (3.16)$$

The horizontal displacement relative to position of firing x at any time t is given by Equation 3.12 where v_h is given by $v \cos \Theta$ and s_h is given by x,

$$x = (v \cos \Theta)\, t \qquad (3.17)$$

and the vertical displacement s_v is given by Equation 3.13b where s_v= y and $v_v = v \sin \Theta$. With these substitutions, Equation 3.13b becomes

$$y = (v \sin \Theta)t + gt^2/2 \qquad (3.18)$$

The vertical displacement $y = 0$ at two times, $t = 0$ and $t = -2\, v \sin \Theta \,/ g$. Notice that this later result is two times the time required to reach the peak. You can also use Equation 3.13b to calculate the peak height.

We can show by substituting the value of t_{peak} given by Equation 3.16 that the peak height y_{peak} is

$$y_{peak} = -v^2 \sin^2 \Theta / g + 1/2 (v \sin\Theta)^2 / g = -1/2\, v^2 \sin^2\Theta / g \qquad (3.19)$$

What is the projectile's range, that is, how far from where it is fired will it strike the ground? If $y = 0$, $x = 0$, at $t = 0$, then the range R is found when y is again zero:

range = horizontal velocity • time of flight

$$R = (v \cos \Theta)(-2\, v \sin \Theta / g) = -v^2 \sin 2\Theta / g \qquad (3.20)$$

In above equations the value of g is -9.80 m/sec^2.

At what angle should the projectile be fired to give maximum range for a given firing velocity? The maximum value of the sin 2Θ occurs when 2Θ is 90°. Hence R is greatest when Θ is 45°. You can obtain the equation for the path of the projectile by combining Equations 3.17 and 3.18 and eliminating t. What is the path of a projectile under ideal conditions?

EXAMPLE

Patty Berg, a professional golfer, drives a ball from the tee with a velocity of 37.2 m/sec (120 ft/sec) at an angle of 37° and with no spin. The fairway is straight, and the ball strikes the ground in the same horizontal plane as the tee. What is the horizontal component of the velocity. How long is the ball in flight? How far down the fairway does the ball first strike the ground? What is the angle at which it strikes the fairway? (Neglect air resistance.)

a. The horizontal component of velocity is $v \cos \Theta$

$$v_h = 37.2 \cos 37° = 29.8 \text{ m/sec or } 96 \text{ ft/sec}$$

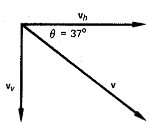


FIGURE 3.10
Graphical representation of the velocity of a projectile at the time it strikes the ground at the end of its flight.

b. The original vertical component of velocity = $v \sin \Theta$.

$$v_v = v \sin \Theta = 37.2 \sin 37° = 37.2 \times 0.6 = 22.3 \text{ m/sec or } 72 \text{ ft/sec}$$

At the peak of the flight the vertical component of velocity is 0. In order to find time to reach the peak, we use Equation 3.15 and set $0 = v \sin \Theta + g t_{peak}$.

$$t_{peak} = -v \sin \Theta / g = -37.2 \times 0.6 / -9.80 = 22.3/9.80 = 2.28 \text{ sec}$$

Total time of flight is equal to two times the time it takes to reach peak, so

$$t_{total} = 2\, t_{peak} = 4.56 \text{ sec.}$$

c. range = $R = v \cos \Theta \bullet t_{total} = 37.2 \times 0.80 \times 4.56 = 136$ m.

d. At the point that the ball hits the ground, the horizontal component of velocity is 29.8 m/sec. We can calculate the vertical component because we know that the ball drops for 2.28 sec.

$$v_v = gt = (-9.8)(2.28) = 22.3 \text{ ft/sec downward}$$

e. Because we now know both the vertical and horizontal components of velocity when the ball strikes the ground, we can find the angle Θ at which it hits.

$$\tan \Theta = v_v / v_h = -22.3/29.8 = -3/4 ;\ \Theta = 37° \text{ below the horizontal.}$$

Is this what you expected? (Look again at Figure 3.9 and at Figure 3.10.)

3.9 Uniform Circular Motion

Do you remember taking a ride on a merry-go-round? This is an example of circular motion. First, consider the ideal situation when the merry-go-round is rotating at a constant rate, that is, moving with constant speed. This is known as *uniform circular motion*. Does a body executing this type of motion have an acceleration? Consider the velocity at two points A and B on the circle in Figure 3.11. At each point the velocity is tangent to the circle at that point. We see that v_A and v_B are not the same since the vector v_A and v_B do not point in the same direction. Hence there must be an acceleration even though the magnitudes of velocity v_A and velocity v_B are equal.

Since the magnitude of the velocity in a direction tangent to the circular path is constant, the value of acceleration in the tangential direction must be zero. Hence, if there is an acceleration, and if the component of acceleration along the direction of motion is zero, the acceleration must be entirely perpendicular to the direction of motion. If the motion is circular, the acceleration must always be directed toward the center of the circle. This acceleration is called the *radial acceleration*.

Let us turn to Figure 3.11 to derive an expression for the magnitude of the radial acceleration. For very small angles the triangles OAB and the velocity triangles in Figure 3.11b are similar triangles. Hence the ratio of the sides is equal,

$$\Delta v / AB = v / r$$

in which line segment AB is the length of the arc from A to B and v represents the magnitude of v_B and v_A. But the distance from A to B is given by the velocity times the change in time, for small time changes, $AB = v \bullet \Delta t$. So,

$$\Delta v / \; v \; \Delta t = v / r \text{ and } \Delta v \; / \; \Delta t = v^2 \; / r$$

But $\Delta v / \Delta t$ is equal to the radial acceleration a_{r}. Hence

$$a_{\mathrm{r}} = v^2 \; / r \qquad (3.21)$$

In the merry-go-round example, the circular motion in starting and stopping is, of course, not uniform.

When the circular motion is not uniform, the tangential component of acceleration is not zero. In these cases, we have both a tangential component of acceleration a_t and a radial component of acceleration a_r. The total acceleration is then the vector sum of the a_t and a_r. The tangential acceleration is positive as the merry-go-round starts and negative as it stops.

EXAMPLE

If you are riding on a merry-go-round at a distance of 6 m from the axis of rotation and are making one revolution in 12 sec, what is your radial acceleration? The tangential velocity of the merry-go-round is the distance traveled, the circumference, divided by the time for one revolution. Thus

$$v = 2 \pi \bullet 6 \text{ m} \bullet 1/12 \text{ sec} = \pi \text{ m/sec}$$

Using Equation 3.21, we can find the radial acceleration.

$$a_r = v^2 \; / r = \pi^2 \; / 6 \approx (10/6) \text{ m/sec}^2$$

What is the tangential acceleration if the merry-go-round reaches this rate of rotation in 6 sec?

$$v_t = a_t \, t$$

Thus

$$\pi = a_t \, t \text{ or } a_t = (\pi/6)\ \text{m/sec}^2$$

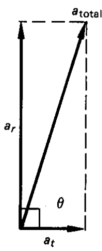


FIGURE 3.12
Graphical representation of the acceleration for a body in circular motion with a variable speed.

Total acceleration (see Figure 3.12) is the resultant of a_r and a_t:

$$a_r = (10/6)\ \text{m/sec}^2 \text{ and } a_t = (\pi/6)\ \text{m/sec}^2$$

$$\tan \Theta = \{(10/6)/6\}/(\pi/6) = 10/\pi$$

$$\Theta = 72.6^\circ$$

$$a_{total} = \sqrt{[(\pi/6)^2 + (10/6)^2]} = 1/6\sqrt{[\pi^2 + (100]} = +1.75\ \text{m/sec}^2$$

during start- up

3.10 Effects of Acceleration

There are examples of acceleration and deceleration within the human body itself. One example has to do with the passage of food through the body. Another is in the blood circulation system. Can you list the positions of acceleration and deceleration for each of these? Can you think of another human system in which there is acceleration and deceleration?

A number of different accelerations may act upon the human body. These vary in duration, magnitude, rate of onset and decline, and direction. Some acceleration exposures may be so mild that they produce no physiological or psychophysiological effects. On the other extreme they may be so severe that they produce major disturbances such as blackouts. The effects also vary a great deal from individual to individual. Undoubtedly, you have observed this in your childhood play and in your reactions to various types of rides at amusement parks.

The field of acceleration research has produced a number of general principles concerning the effects of acceleration on human physiology and performance. For additional details and information see the *Bioastronautics Data Book*, Scientific and Technical Information Division, National Aeronautics and Space Administration, from which is derived Table 3.4 showing the effects on humans due to sustained acceleration.

FIGURE 3.13
Sketch of human being accelerated (a) upward, (b) downward, (c) in forward direction, and (d) in a backward direction.

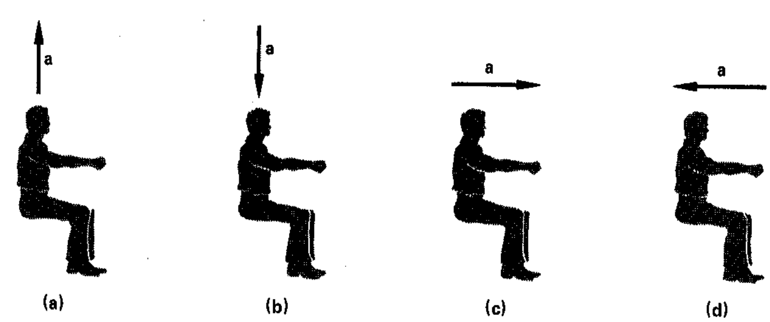


TABLE 3.4
Effects of Acceleration on the Human Body

Type of Acceleration	Magnitude in Units of $g = 9.8$ m/sec^2	Effect
Positive vertical acceleration (up) (Figure 3.13a)	$1g$	Normal condition at sea level
	$2.5g$	Difficult to raise oneself
	$3g$ to $4g$	Impossible to raise oneself; difficult to raise arms and legs; vision dims after 3-4 sec
	$4.5g$ to $6g$	Diminution of vision; progressive to blackout after 5 sec
Negative vertical acceleration (down) (Figure 3.13b)	$-1g$	Unpleasant facial congestion
	$-2g$ to $-3g$	Severe facial congestion; throbbing headache, and blurring vision
Forward horizontal acceleration (Figure 3.13c)	$2g$ to $3g$	Progressive difficulty in focusing and slight spatial disorientation (2 g tolerable for at least 24 hours)
	$3g$ to $6g$	Progressive tightness in chest; loss of peripheral vision; blurring of vision; difficulty in breathing and speaking
Backward horizontal acceleration (Figure 3.13d)	$2g$ to $3g$ $3g$ to $6g$	Similar effects to forward acceleration with the modification that chest pressure is reduced and breathing is easier

EXAMPLE

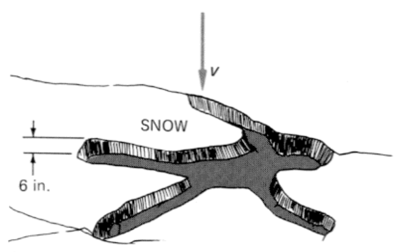


FIGURE 3.14

Find the stopping acceleration (average) in units of g for a person striking a snow drift at the terminal velocity of 54 m/sec if 1 m of snow brings the person to rest Figure 3.14. From Equation 3.9,

$$v_f^2 - v_o^2 = 2a \bullet s$$

Substituting the given values of $v_f = 0$, and $v_o = 54$ m/sec, $a = -(54 \text{ m/sec})^2/2 \text{ m} = -1438.6 \text{ m/sec}^2 = 146.8\ g$ There is a documented case of a paratrooper free falling without a chute and surviving such a fall without major injuries!

ENRICHMENT

3.11 Instantaneous Velocity and Acceleration

In Equation 3.1 we defined average velocity $v_{ave} = \Delta s/\Delta t$. As the change in time approaches zero ($\Delta t \rightarrow 0$), the instantaneous velocity is the limiting value of $\Delta s/\Delta t$ and is written ds/dt. This is called the derivative of s with respect to t,

$$v_{inst} = \lim_{\Delta t \rightarrow 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt} \qquad (3.22)$$

Thus if s = f(t), then

$$v_{inst} = \frac{ds}{dt} = \frac{df(t)}{dt} = f'(t) \qquad (3.23)$$

Similarly average acceleration is $a_{ave} = \Delta v/\Delta t$, and the instantaneous acceleration is

$$a_{inst} = \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d^2 s}{dt}$$

EXAMPLES

1. Suppose a body moves along the x-axis in accord with the relationship

$$x = 4 - 3t + 2t^2 \text{ meters}$$

Find the instantaneous velocity, instantaneous acceleration, and when the body is at rest.

$$v_{inst} = dx/dt = -3 + 4t \text{ m/sec} = v_o + at$$

a. Explain this relationship; that is, what do the -3 and 4 represent? [the initial velocity and the instantaneous acceleration]

b. What does the fact that $a_{inst} = 4$ m/sec^2 indicate about the motion of this body? [It is uniformly accelerated motion.]

c. For a body at rest $v_{inst} = 0$. When does this occur? [when $-3 + 4t = 0$ or $t = 3/4$ sec]

2. Develop the equation of motion of a particle from the following information: An object starts from $y = 2$ m with a velocity of 3 m/sec and a constant acceleration of -0.5 m/sec^2.

$$a = dv/dt = -0.5 \text{ m/}^2$$

Thus, $$\int dv = \int a\, dt$$

so v= -0.5t + constant. At t = 0, v = +3 m/sec. So the constant is 3 m/sec. What is v at any time t? v = -0.5 t + 3 and v= dy/dt, so

$$\int dy = \int v\, dt$$

What is the value y at any time t? [$y = -(0.5t^2)/2 + 3t$ + constant. y = 2 at t = 0, so the constant is 2 m. $y = 2 + 3t - 0.25t^2$.]

SUMMARY

Use these questions to evaluate how well you have achieved the goals of this chapter. The answers to these questions are given at the end of this summary with the section number where you can find the related content material.

Definitions

1. The slope of a displacement versus time curve is called _____.
2. The speed of an object is constant when it undergoes _____.
3. Uniform circular motion implies that _____is always zero.
4. If the _____is constant, then uniformly accelerated motion is observed.
5. The idealized motion of projectiles near the surface of the earth has a constant _____ in horizontal direction and a constant ______ in vertical direction.
6. When a fly wheel on an electric motor starts, it has a positive _____ and its motion is not uniform.
7. An object moving with constant speed around a circle has_____ acceleration.

Equations of Motion

From the equations of uniformly accelerated motion (see Section 3.7) you should be able to answer the following questions.

8. An object whose motion is described as uniformly accelerated always has which of the following properties?
    a. the speed is constant
    b. acceleration is proportional to time
    c. displacement is a quadratic function of the time
    d. the velocity vector does not change its direction

9. The velocity of a uniformly accelerated bicycle
    a. increases linearly with distance
    b. increases linearly with time
    c. increases linearly with acceleration
    d. increases linearly with gravitation

10. For uniformly accelerated motion which of the following quantities must be zero?
    a. the initial acceleration
    b. the initial velocity
    c. the initial displacement
    d. the time rate of change of the acceleration
    e. the time rate of change of the velocity
    f. the time rate of change of the displacement

Motion Problems

From the equations in Section 3.5 you can solve problems about idealized freely falling objects.

11. A parachutist jumped from a helicopter at rest at a height of 78.4 m above the ground. His parachute failed to open. Neglecting air resistance, how long did it take him to hit the ground, and what was his speed at impact?
    a. 2 sec, 4.9 m/sec
    b. 4 sec, 39.2 m/sec
    c. 6 sec, 58.8 m/sec
    d. 4 sec, 39.2 m/sec
    e. 2 sec, 9.8 m/sec From the equations in Section 3.8 you can solve problems of projectile motion.

12. A swimmer leaps horizontally from the edge of the swimming pool with a velocity of 6 m/sec. If he is 2.4 m above the surface of the water when he leaves the edge of the pool, assuming idealized motion, how long will it be before he hits the water? How far will he be from the edge of the pool?
    a. 0.9 sec, 5.4 m
    b. 0.8 sec, 4.8 m
    c. 0.7 sec, 4.2 m
    d. 0.6 sec, 3.6 m
    e. 0.5 sec, 3.0 m

From the equations in Section 3.9, you can solve problems on uniform circular motion.

13. During pre-mission training, the astronauts are placed in the large NASA centrifuge. They are swung in a horizontal circle at a constant speed of 10 m/sec on the end of a 3.4 m long support rod. What is the horizontal acceleration the astronauts must endure?
    a. 4.9 m/sec^2 or ½ g

b. 9.8 m/sec^2 or 1 g
c. 19.6 m/sec^2 or 2 g
d. 29.4 m/sec^2 or 3g
e. 39.2 m/sec^2 or 4 g

Acceleration Effects

14. From your reading of Section 3.10 describe the acceleration effects on a space shuttle transporter occupant
    a. when it takes off with a 3 g acceleration
    b. when it lands with a 2 g braking acceleration

Answers

1. velocity (Section 3.6)
2. uniform circular motion (Section 3.9)
3. tangential acceleration (Section 3.9)
4. acceleration (Section 3.7)
5. velocity, downward acceleration (Section 3.8)
6. tangential acceleration (Section 3.9)
7. radial (Section 3.9)
8. c (Section 3.7)
9. b, c (Section 3.7)
10. d (Section 3.7)
11. d (Section 3.5)
12. c (Section 3.9)
13. d (Section 3.10)
14. a. impossible to raise oneself, vision dims (Section 3.10)
    b. facial congestion

ALGORITHMIC PROBLEMS

Listed below are the important equations from this chapter. The problems following the equations will help you learn to translate words into equations and to solve single concept problems.

Equations

$$v_{ave}=(s_1-s_0)/(t_1-t_0)=\Delta s/\Delta t \qquad (3.1)$$

$$s = v_{ave}\, t \qquad (3.2)$$

$$a_{ave} = \Delta v/\Delta t = (v_f - v_0)/t \qquad (3.3)$$

$$v_f = v_0 + at \qquad (3.4)$$

$$v_{ave} = (v_f+v_0)/2 \qquad (3.5)$$

$$\Delta s = s - s_0 = v_{ave}\, t = (v_f+v_0)/2)t \qquad (3.6)$$

$$s - s_0 = v_0 t + at^2/2 \qquad (3.7)$$

$$2a \bullet s = v_f^2 - v_o^2 \qquad (3.9)$$

$$v_v = v \sin \Theta + gt \qquad (3.14)$$

$v_h = v \cos \Theta$ (definition)

$x = (v \cos \Theta)\, t$ (3.17)

$y = (v \sin \Theta)t + gt^2/2$ (3.18)

$y_{peak} = -(1/2)\, v^2 \sin^2\Theta / g$ (3.19)

$R = -\, v^2 \sin 2\Theta / g$ (3.20)

$v_t = 2\, \pi r n$ (definition)

Problems

1. Find the average speed of a sprinter who runs 100 m in 9.1 sec.
2. If a skier reaches a speed of 20 m/sec in 10 sec after starting from rest, find the acceleration of the skier.
3. A ball is dropped from a window 19.6 m above the ground. Assuming idealized motion, how long does it take the ball to reach the ground?
4. An experimental bumper system is designed to bring a car to rest from an initial speed of 4.0 m/sec. The stopping distance of the bumper is 0.50 m. Find the negative acceleration necessary to make such a stop.
5. A train is originally moving at a speed of 20.0 m/sec when it is accelerated at 2.00 m/sec^2 for 5.00 sec. Find the distance the train travels during the time of acceleration.
6. A toy train goes around a circular track (radius 1.00 m) at a constant speed of 1.50 m/sec. Find the radial acceleration of the train.
7. The wheel of a moving bicycle is 71.1 cm in diameter and is making 2.00 revolutions per second. Assume that the wheel does not slip on the ground. How fast is the bicycle traveling?
8. A student hits a ping-pong ball at the back edge of the table so that the ball leaves the paddle with a velocity of 2.0 m/sec at 30° above the horizontal. Assume idealized projectile motion. What is the horizontal velocity as the ball leaves the paddle? When is the velocity of the ball entirely horizontal?
9. An object is moving along a straight line such that its displacement is as shown below. What is the average velocity for each second and the entire 3 seconds? See Table, where *x* is given in meters and *t* is in seconds

x	t	v_{ave}
0	0	______
−2	1	______
0	2	______
12	3	______

10. A golf ball is projected at 45° to the horizontal with an initial velocity of 40 m/sec. a. Find the horizontal and vertical speed of the ball 5.0 sec after it is projected. b. Find the horizontal and vertical position of the ball after the first 5.0 sec of flight.
11. Find the peak height of the golf ball in the flight described in problem 10.
12. Find the initial velocity of a projectile launched at an angle of 30°, if its peak height is 25 m.
13. Compute the tangential velocity and the radial acceleration of an object resting on the edge of a long playing phonograph record (r = 15 cm, n = 33 1/3 rpm).

Answers

1. 11 m/sec^2
2 m/sec^2
3. 2.00 sec
4. 16 m/sec^2
5. 125 m
6. 2.25 m/sec^2
7. 447 cm/sec
8. 1.7 m/sec, .010 sec
9. -2 m/sec, 2 m/sec, 12 m/sec, 4 m/sec
10. a. 28 m/sec, -21 m/sec
    b. 140 m, 19 m
11. 41 m
12. 44 m/sec
13. 52 cm/sec, 180 cm/sec^2

EXERCISES

These exercises are designed to help you apply the ideas of a section to physical situations. When appropriate, the numerical answer is given in square brackets at the end of each exercise.

Section 3.2

1. A body undergoes the following displacements: 6 m in northwest direction, 10 m at an angle of 37° south of west, and 12 m at angle 30° south of east. What is the final position of the body relative to the original position? [8.0 m, 260°]
2. The weight of a body is a vector quantity, and its direction is vertically downward. If a block of marble weighing 500 newtons (N) is resting on a 20° incline, what are the components of the weight parallel to the incline and perpendicular to the incline? [171 N, 470 N]

Section 3.6

3. A city bus starts from rest at a bus stop and accelerates at the rate of 4.0 m/sec^2 for 10 sec. It then runs at this constant rate for 30 seconds and decelerates at 8.0 m/sec^2 until it stops. Draw a graph of the displacement versus time. What is the displacement of the bus between stops? [1500 m]

Section 3.7

4. A geology student is trying to determine the depth of a ravine by dropping rocks from a cross-walk. He finds by a stopwatch that 2.50 sec is required for a rock dropped from the bridge to strike the water. Assuming idealized motion, how deep is the ravine? [30.6 m]
5. The reaction time of an alert automobile driver is 0.700 sec. (The reaction time is the interval between stimulus to stop and application of brakes.) After application of the brakes an automobile can decelerate at 4.9 m/sec^2. If a car is traveling at 48.4 km/hr (30 mph), what total distance does it travel after the driver sees a stop signal? How far does it travel if the car is traveling with a velocity of 96.8 km/hr (60 mph)? Does this seem realistic to you? What difference would it make if the driver had been drinking and had a slower reaction time of 1.50 sec? [27.8 m, 92.5 m, 10.8 m farther, 21.5 m farther]
6. There are cases in which the human body has withstood very large accelerations under proper conditions. The following is based on an actual incident. A female, age 21, height 1.7 m (5 ft 7 in.), mass 52.3 kg (weight 115 lbs), jumped from a tenth-story window and fell 28.4 m (93 ft) into a freshly plowed garden where she came to rest with a deceleration distance of 15.3 cm (6 in.). She landed on her right side and back, and her head struck the soft earth (see Figure 3.14). The woman survived, sustaining

only a fractured rib and right wrist. Apparently there was no loss of consciousness or concussion. Assume a freely falling body. What were the velocity of impact and the deceleration in g's? [23.6 m/sec, 92.7 g]

Section 3.8

7. A ping-pong ball rolls with a speed of 0.60 m/sec toward the edge of a table top which is 0.80 m above the floor. The ball rolls off the table. Assuming idealized motion, how long was it in flight, and how far from the edge of the table did the ball hit the floor? [0.40 sec, 0.24 m]
8. An aviator drops a heavy object from his plane at a height of 490 m while he is moving with a constant horizontal velocity of 30 m/sec. How long does it take for the object to strikes the ground? Where is the plane when the object strikes the ground? Where does the object strike the ground relative to the point directly under the plane at the instant the object was dropped? [Neglecting friction, 10 sec, plane vertically above object, 300 m]
9. A baseball leaves the bat of Hank Aaron at a height of 1.22 m (4 ft) above the ground at an angle of 37° with such velocity that it would have a range of 122 m at the height of 1.22 m. However, at a distance of 106.7 m (350 ft) from homeplate there is a 9.15 m (30 ft) high fence. Does Aaron get a home run? [yes, ball is higher than 9.15 m at 106.7 m]
10. A punter kicks a football at an angle of 53° above horizontal. It is observed to be in the air 4 sec. How high did it go, and how long was the kick? Would you want this player for a punter on your football team? [19.6 m, 58.9 m or about 64 yards]

Section 3.9

11. An aviator is said to be doing a 4g circle. What does this mean? What kind of a circle would he be doing if the resultant acceleration at the top of the circle is 0? [v^2 /r = 4 g, 1g = v^2 /r]
12. At what speed must an automobile round a curve with a radius of curvature of 39.2 m to have a radial acceleration equal to g? Now suppose the car continues down the road at the same speed and goes over the top of a hill with the same radius of curvature. What sensation would you experience if you were a passenger in the car? [19.6 m/s]
13. As a result of the earth's rotation, objects at rest on the surface of the earth have a radial acceleration. What is this acceleration at the equator? The radius of the earth is 6.38×10^6 m. What is this acceleration at your latitude? How does this compare with g? [$a_{\text{equator}} = 3.37 \times 10^{-2}$ m/sec^2 = $3.45 \times 10^{-3}\, g$]

PROBLEMS

Each problem may involve more than one physical concept. A problem that requires material from the enrichment section, Section 3.11, is marked by a dagger (†). The numerical answer is given at the end of each problem.

14. A baseball outfielder can throw a baseball a maximum distance of 78.4 m over the ground before reaching the height from which it was thrown. Assuming idealized motion, with what velocity does he throw it? How long will the ball be in flight? How many bases can a runner safely take during this time if he can run the 100-meter dash in 11 sec? The distance between bases is 27.4 m or 90 ft. [27.7 m/sec, 4.00 sec, 1 base]

15. An open automobile starts from rest with a uniform acceleration at 4.00 m/sec^2. A premed student stands on the other side of the parking lot, exactly opposite the starting point of the car and 20.0 m from it. As it starts, he throws an apple with a horizontal velocity of 20.0 m/sec to a classmate in the car. In what direction must he throw the apple so his classmate can catch it easily? Assume it passes over the shortest possible distance. about 5.75° to the line between the student and the original position of the car]
16. A small rocket is shot vertically into the air with a speed of 300 m/sec. In addition to the acceleration due to gravity there is an average retarding acceleration of 2.20 m/sec^2. How long does it take for the rocket to reach maximum height? What is this height? [2.50 sec, 37.5 m]
17. A helicopter is ascending at a constant rate of 15.0 m/sec. A doctor drops a weighted package of bandages from the helicopter at a height of 60.0 m to a nurse below. What is the time of flight of the bandages as observed by the nurse on the ground? [5.35 sec]
18. A med-tech student drops a stone from a bridge 19.6 meters above a river. A premed friend throws a stone 0.500 sec later vertically downward so that both stones strike the river at the same time. With what velocity did the premed throw the stone? [5.73 m/sec]
19. Superphysicist (SP) dives out of a window h meters above the ground to save a freely falling sky diver whose chute failed to open. He leaves the window horizontally when his laser eyes see the diver at the same level as the window at distance of D meters away. If the diver fell from a plane at an altitude of H with a velocity v (m/sec) directed horizontally away from SP's building, find the average horizontal velocity SP must have to catch the diver at ground level(see Figure 3.15). [$v_{ave} = v + Dg/[\sqrt{(2gH)} - \sqrt{(2g(H - h))}]$] (What is the physical meaning of $\sqrt{(2gH)}$ and $\sqrt{(2g(H\text{-}h))}$?

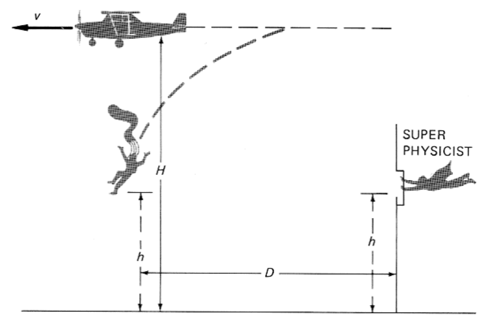


FIGURE 3.15
Problem 19.

20. A well-conditioned astronaut can jump 1.5 m on earth. The gravitational acceleration on the moon is one-sixth that of the earth. Find:
 a. his initial vertical velocity when jumping on the earth
 b. his initial vertical velocity when jumping on the moon
 c. how high he jumped on the moon [v = 5.4 m/sec, v = 5.4 m/sec, 9.0 m]
21. A slingshot launches a projectile with speed v at an angle Θ with respect to the ground. Find the velocity of the projectile at the top of its path. Find the acceleration at the top of its path. Since acceleration is perpendicular to velocity at the top, the motion is "instantaneously circular." Find the radius of this circular arc. [$v \cos\Theta$, g down, $v^2 \cos^2\Theta / g$]
22. A river is flowing south 3.0 km/hour. A canoer can paddle 4.0 km/hr relative to water. a. Where must he head his canoe if he wants to go across the river in a direction perpendicular to the bank? b. How long will it take him to cross the river if it is one-half kilometer wide? c. If he wishes to cross the river in minimum time, where should he head the canoe? d. How long will it take him to cross the river, and where would he land? [a. 41° with bank upstream; b. 0.19 hr; c. perpendicular to

bank; d. 1/8 hr, 3/8 km downstream]

23. Suppose a ferris wheel with a radius of 9.6 m and a constant tangential speed of 10 m/sec loses a chair at the top of its path. Find the horizontal distance the chair will travel before hitting the ground. Assume center of ferris wheel is 10 m from the ground. [20 m]
24. Given the velocity-time graph in Figure 3.16 for the 400-m run,
 a. find the maximum acceleration for this run.
 b. find the distance traveled during the positive and negative acceleration periods.

 [a. 5.00 m/sec^2;
 b. 10.0 m during positive acceleration;
 18.0 m during negative acceleration]

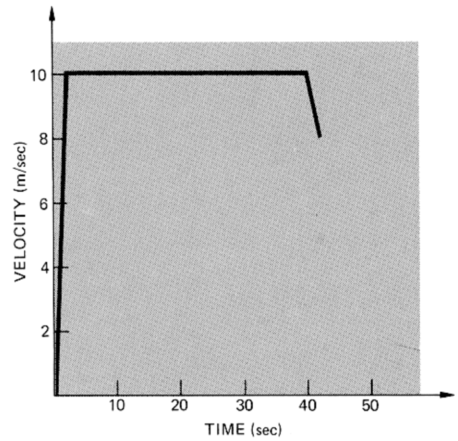


FIGURE 3.16

25. What is the component of the gravitational acceleration parallel to an inclined plane 13 m long that is elevated 5.0 m at one end. Neglecting friction, how long would it take a body starting from rest to slide 9.8 m down the incline? [3.8 m/^{2}, 2.3 sec]

†26. For a particle whose position is given by $x = 2.00\ t^2 - (25.0/300)t^3$ as a function of time, t, a. Find the velocity and acceleration as a function of time. b. Find the maximum value of x. [$v = 4.00\ t - 0.250\ t^2$, a = 4.00-0.500 t; $x_{max} = 171$]

†27. $x = 10 + 20\ t^2 - 30\ t^4$ is the position of a particle as a function of time, where x is in meters and t is in seconds.
 a. Find the velocity and acceleration as a function of time.
 b. Find the time when acceleration is zero.
 c. Find the maximum value of x.
 [$v = 40t - 120\ t^3$, $a = 40 - 360\ t^2$, $a = 0$ at $t = 1/3$ sec, $s_{max} = 13\ 1/3$ m]

†28. A body moves along the x-axis according to the relationship $s = 4 - 6\ t + 3\ t^2$ cm where t is the time in seconds.
 a. Where is the body at $t = 0$?
 b. Where is the body at $t = 2$?
 c. What is the original velocity?
 d. What is the velocity at time $t = 3$?
 e. When is the body at rest?
 f. Where is the body when it is at rest?
 g. What is the acceleration of the body?
 h. Through what distance did the body travel between t = 0.5 sec and t = 2 sec?
 [a. 4 cm; b. 4 cm; c. -6 cm/sec; d. 12 cm/sec; e. 1 sec; f. 1 cm; g. 6 cm/sec^2; h. 3.75 cm]